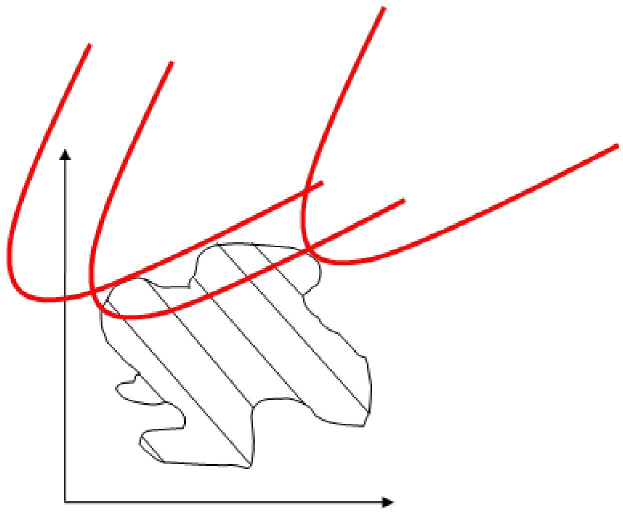
Operations Research



HYUNSOO LEE

Announcements

- Mid-term schedule
  - April 23rd (Monday)
  - Bring a "engineering calculator"
    - Inverse Matrix
  - "Closed-book"

Review (1)

- Mathematical Programming
  - Objective function
  - Constraint
  - Non-negativity

- Types of Mathematical Programming
  - Linear Programming
  - Non-Linear Programming
  - Stochastic Programming
  - Dynamic Programming

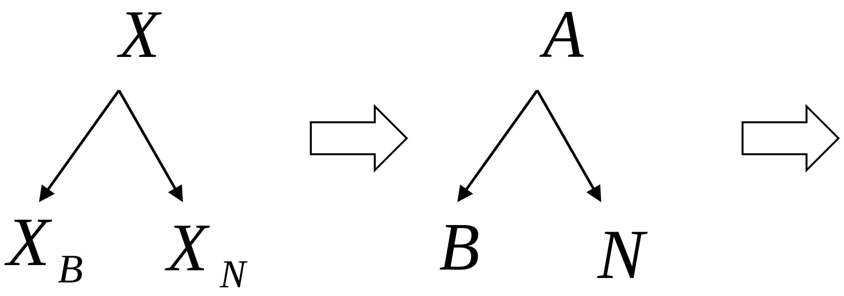
Review (2)

- Linear Programming

$$\begin{aligned} &Min \quad CX \\ &s.t. \quad AX \le b \\ &\qquad\quad X \ge 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} &Min \quad CX \\ &s.t. \quad AX = b \\ &\qquad\quad X \ge 0 \end{aligned}$$

Review (3)

- AX = b



$$BX_B + NX_N = b$$

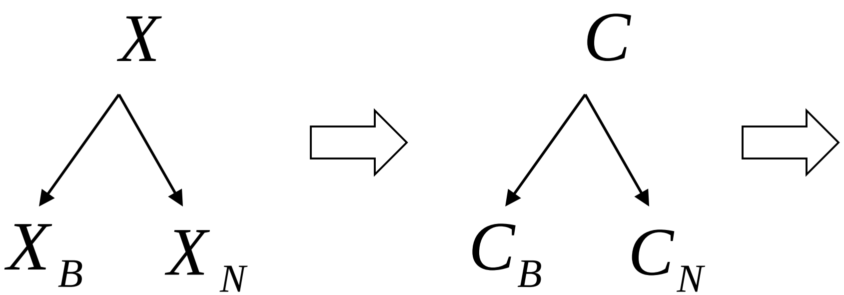
$$BX_B = b - NX_N$$

$$X_B = B^{-1}(b - NX_N)$$

$$X_B = B^{-1}b - B^{-1}NX_N$$

Review (4)

- Min CX



$$C_B X_B + C_N X_N$$

$$C_B(B^{-1}b - B^{-1}NX_N) + C_N X_N$$

$$C_B B^{-1}b - (C_B B^{-1}N - C_N)X_N$$

Review (5)

- Transition to another corner point

$$X_B \Rightarrow X_N$$

$$CX$$

$$X_N \Rightarrow X_B$$

$$C_B B^{-1} b - (C_B B^{-1} N - C_N) X_N$$

$$(C_B B^{-1} N - C_N)$$

Example (1)

- Possible Question in Mid-term

$$\begin{aligned} Max \quad & 2x_1 - x_2 \\ s.t. \quad & 2x_1 + 3x_2 = 6 \\ & x_1 - 2x_2 \le 0 \\ & x_1, x_2 \ge 0 \end{aligned}$$

  - Question :
    - When X_1 is non-basic variable, What is the objective value?

Example (2)

- Possible question in Mid-term

$$\begin{aligned} Max \quad & 11x_1 + 2x_2 - x_3 + 3x_4 + 4x_5 + x_6 \\ s.t. \quad & 5x_1 + x_2 - x_3 + 2x_4 + x_5 = 12 \\ & -14x_1 - 3x_2 + 3x_3 - 5x_4 + x_6 = 2 \\ & 2x_1 + 0.5x_2 - 0.5x_3 + 0.5x_4 \le 2.5 \\ & 3x_1 + 0.5x_2 + 0.5x_3 + 1.5x_4 \le 3 \\ & x_1, x_2, x_3, x_4, x_5, x_6 \ge 0 \end{aligned}$$

  - Question :
    - When X_1 , X_3 , X_5 and X_8 are non-basic variable, What is the objective value?

Summary of Linear Programming

- In L.P.
  - Can you model a Problem?
  - What's your first Basic solution?
  - What's your objective function value?

- Limitations of L.P.
  - Two limitations

Multi-Objective Linear Programming

- M.O.L.P

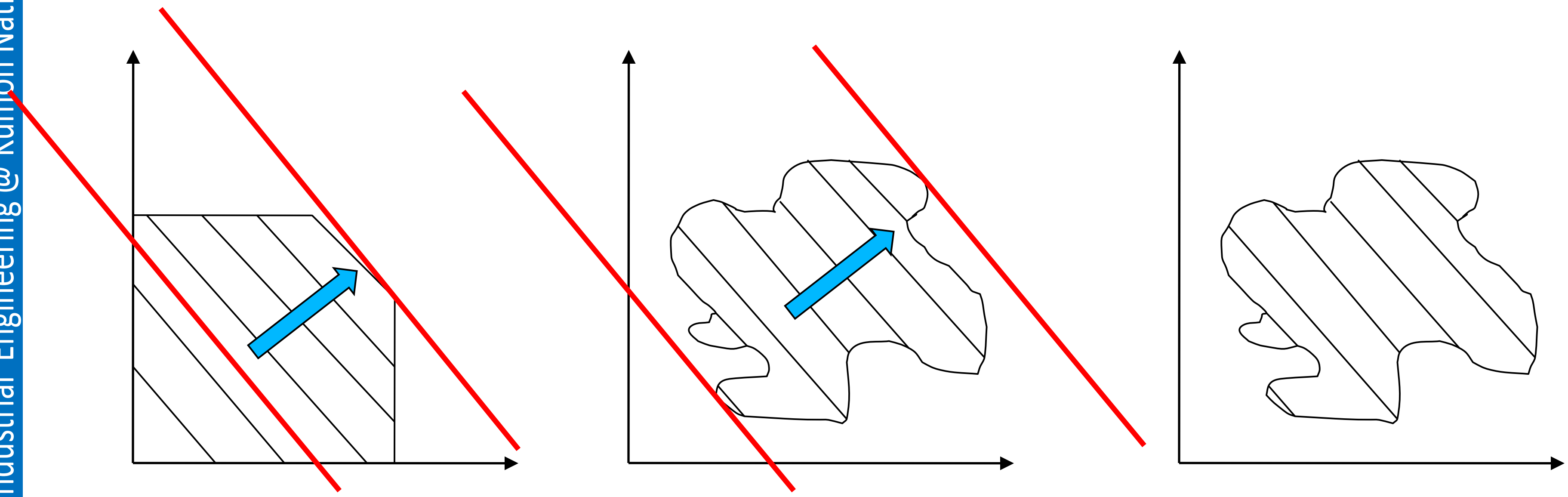
$$Max \quad 11x_1 + 2x_2 - x_3 + 3x_4 + 4x_5 + x_6$$

$$Min \quad 5x_1 + 100x_2 + x_3 - 3x_4 - 54x_5$$

$$
\begin{aligned}
s.t. \quad & 5x_1 + x_2 - x_3 + 2x_4 + x_5 = 12 \\
& -14x_1 - 3x_2 + 3x_3 - 5x_4 + x_6 = 2 \\
& 2x_1 + 0.5x_2 - 0.5x_3 + 0.5x_4 \le 2.5 \\
& 3x_1 + 0.5x_2 + 0.5x_3 + 1.5x_4 \le 3 \\
& x_1, x_2, x_3, x_4, x_5, x_6 \ge 0
\end{aligned}
$$

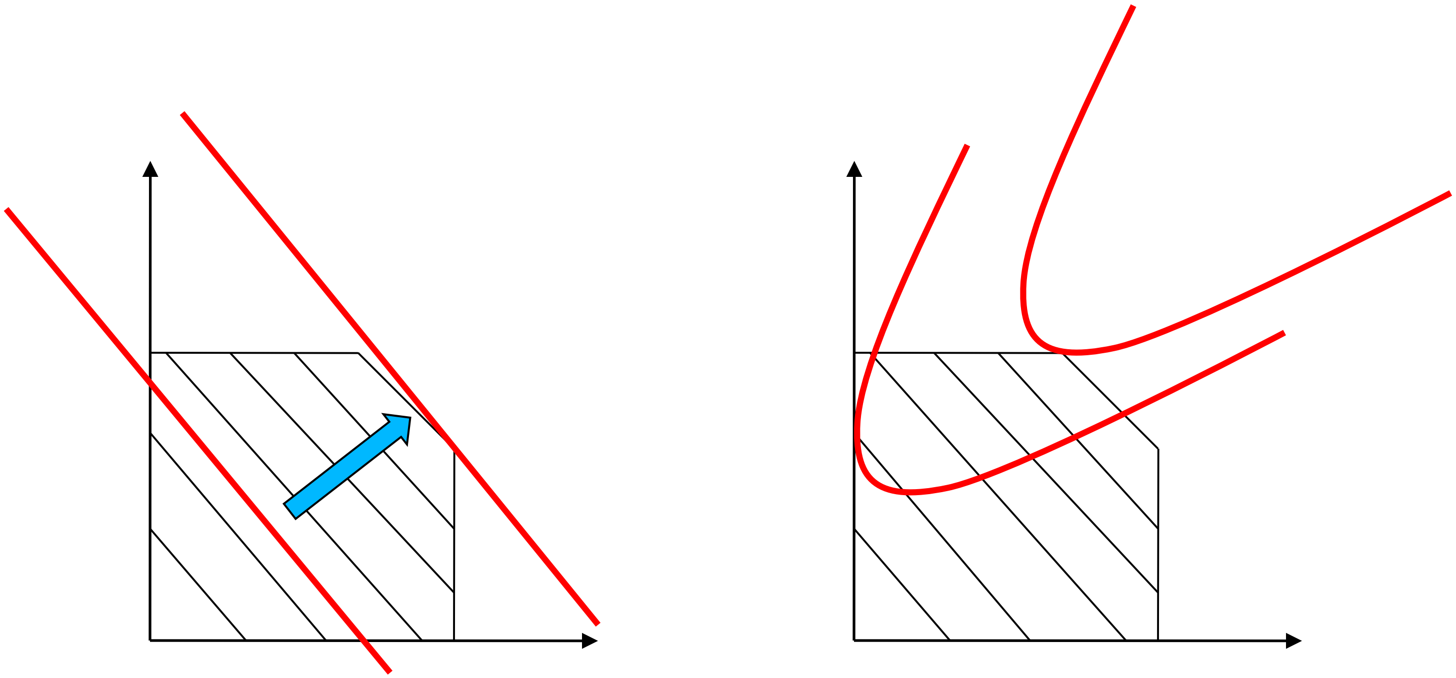
Part 2. Non-Linear Programming (1)

- Case 1



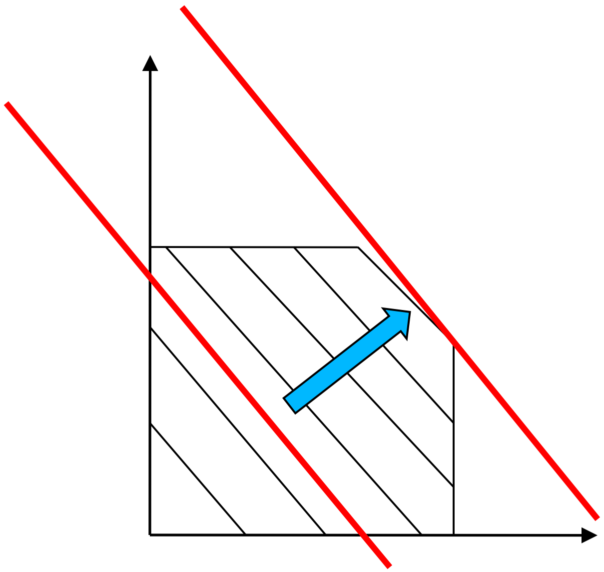
Part 2. Non-Linear Programming (2)

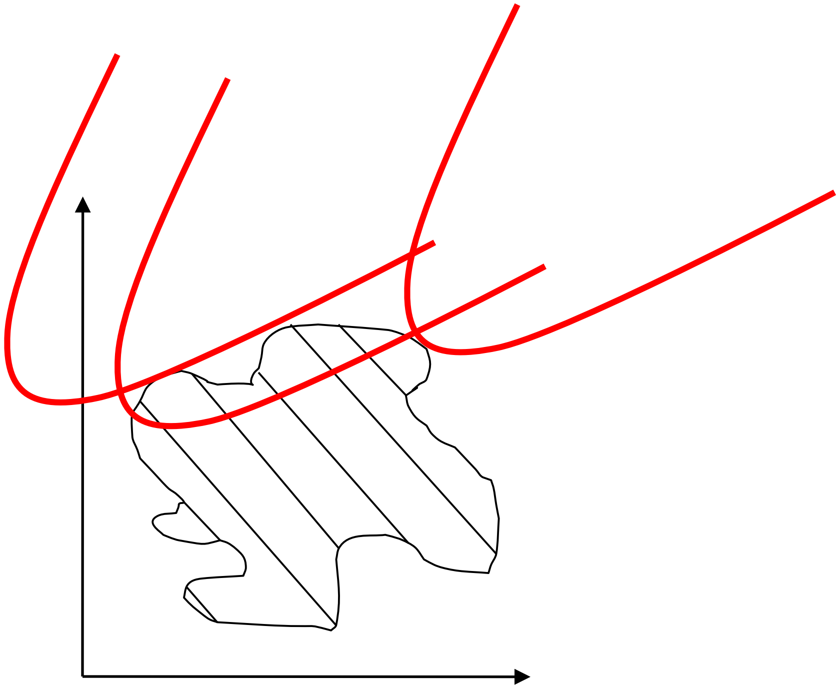
- Case 2



Part 2. Non-Linear Programming (3)

- Case 3



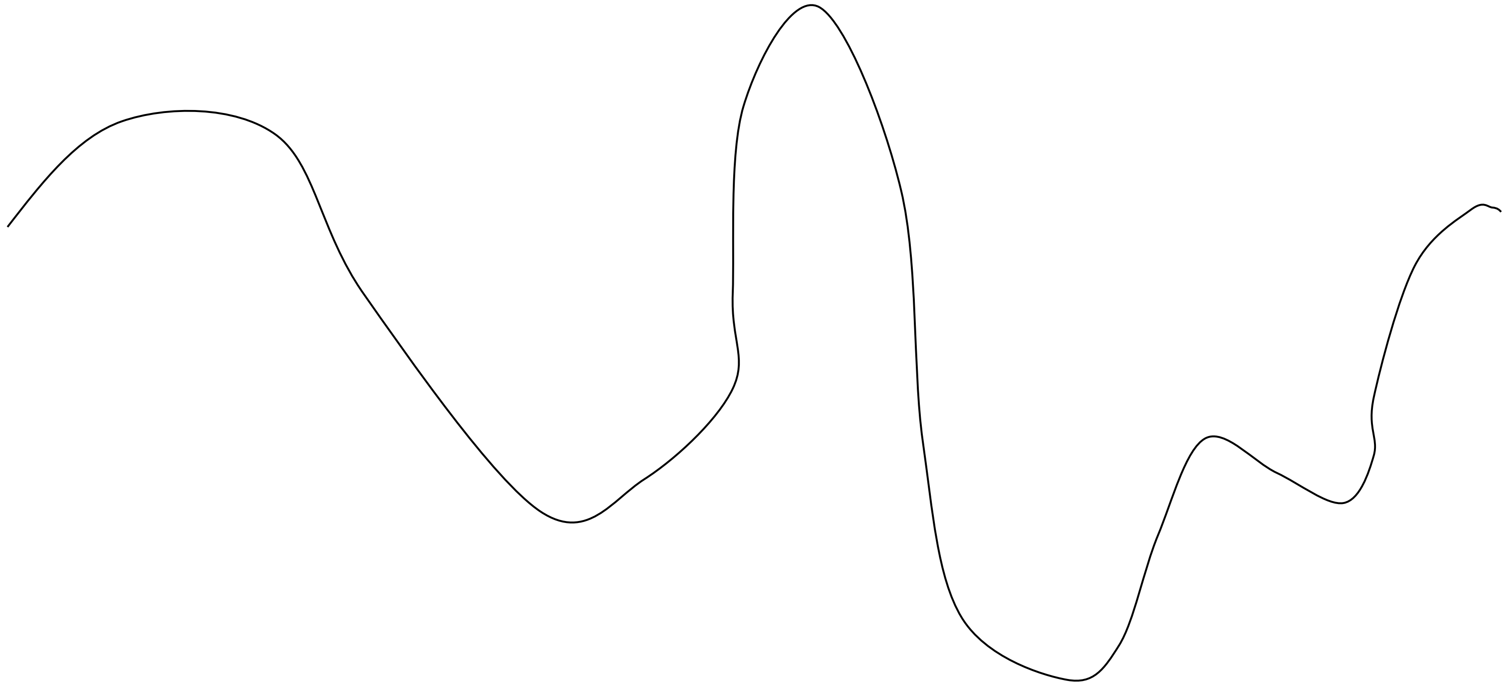


Basic Idea of N.L.P (1)

- Basic Idea
  - Starting point → Searching

Basic Idea of N.L.P (2)

- Local Optimum V.S. Global Optimum



Basic Idea of N.L.P (3)

- Local Optimum (In Minimization)

$$|x - x^*| \le \varepsilon \qquad\qquad f(x) \ge f(x^*)$$

- Global Optimum (In Minimization)

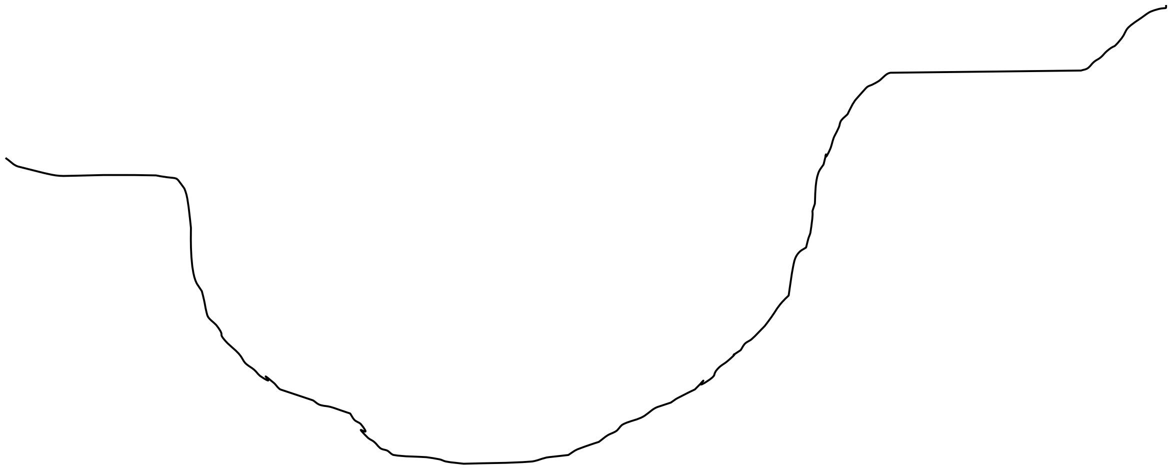
$$f(x) \ge f(x^*)$$

Types of N.L.P

- Types
  - Unconstraint N.L.P
    - Objective function
  - Constraint N.L.P
    - Objective function
    - Constraint
    - (Non-negativity)

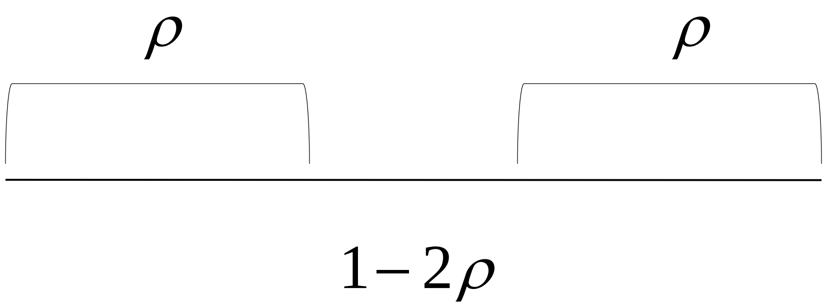
Unconstraint N.L.P (1)

- One variable case
  - One dimensional Search case



Unconstraint N.L.P (2)

- One variable case



$$1:\rho = 1-\rho : 1-2\rho$$

$$\rho(1-\rho) = 1-2\rho$$

$$\rho^2 - 3\rho + 1 = 0$$

$$\rho = 0.382 \;\; and \;\; 0.628$$

Unconstraint N.L.P (3)

- Golden search

$$Min \quad x_1^4 - 14x_1^3 + 60x_1^2 - 70x_2$$

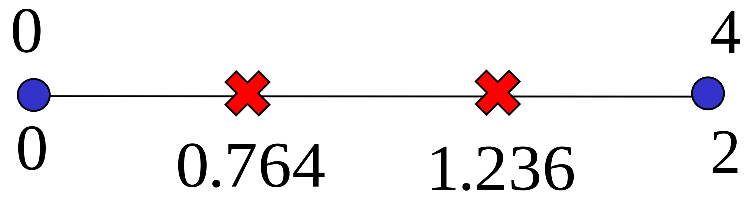
  - Range = [0,2] , Error interval = 0.3

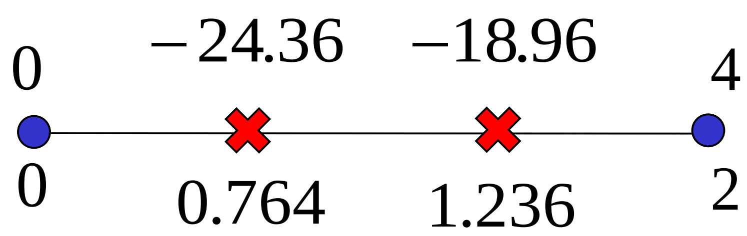
0 4

0 2

Unconstraint N.L.P (4)

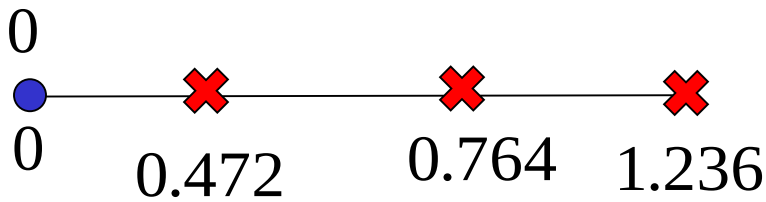
$$Min \quad x_1^4 - 14x_1^3 + 60x_1^2 - 70x_2$$

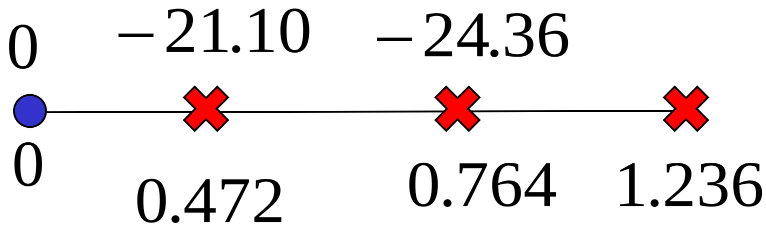




Unconstraint N.L.P (5)

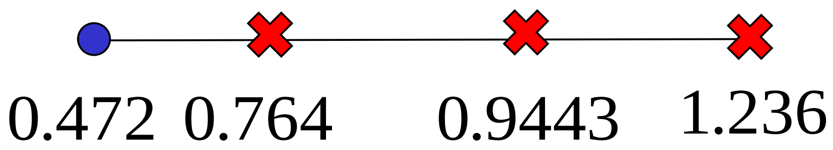
$$Min \quad x_1^4 - 14x_1^3 + 60x_1^2 - 70x_2$$

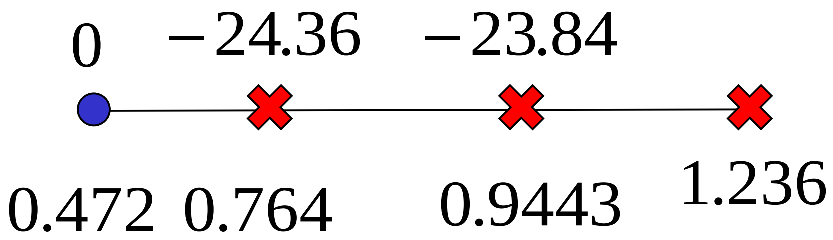




Unconstraint N.L.P (6)

$$Min \quad x_1^4 - 14x_1^3 + 60x_1^2 - 70x_2$$





Unconstraint N.L.P (7)

- Exercise

$$Min \quad x_1^4 - 17x_1^3 + 58x_1^2 - 60x_2$$

  - Range = [1,2] , error interval = 0.1

Unconstraint N.L.P (8)

- Multi-variable case

$$Min \quad x_1^2 + x_2^2$$

  - Starting point → (2,1)

Unconstraint N.L.P (9)

- Multi-variable case

$$(2x_1, 2x_2)$$

$$(4,2)$$

  - Then ,

$$(2-\alpha 4, 1-\alpha 2)$$

$$Min \quad x_1^2 + x_2^2$$

Unconstraint N.L.P (10)

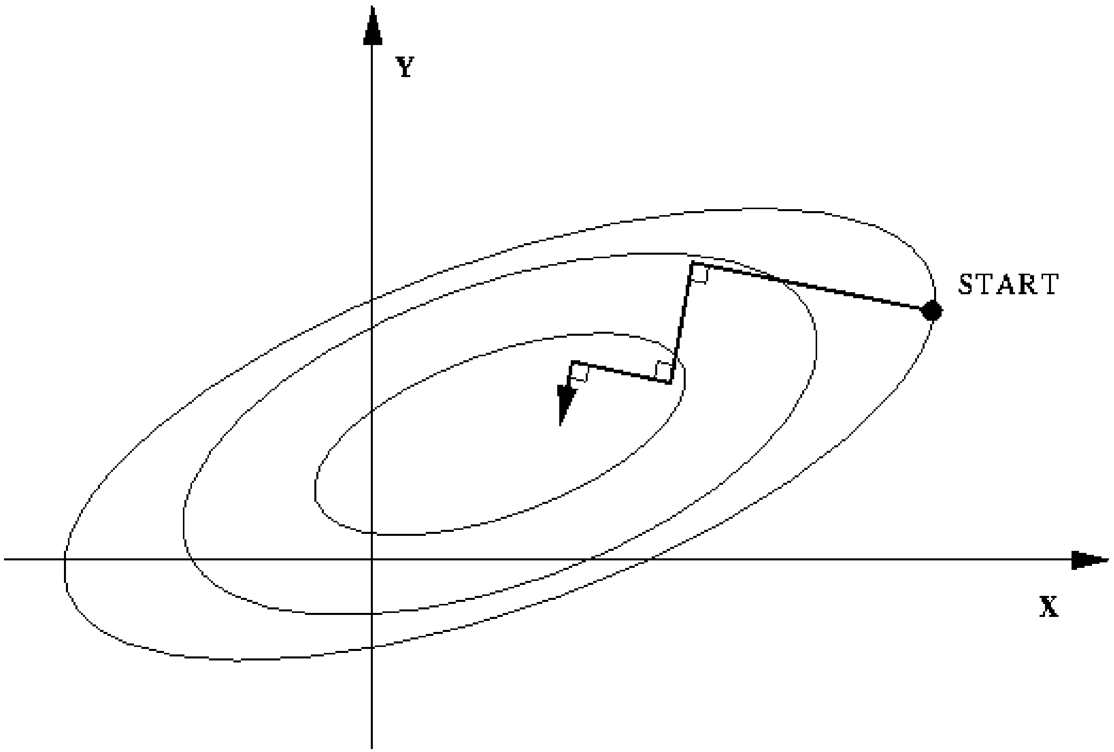
- Example

$$Min \quad (x_1 - 4)^4 + (x_2 - 3)^2 + 4(x_3 + 5)^4$$

  - Initial point = [4, 2, -1]

Unconstraint N.L.P (11)

- Steepest Descent Method



Constrained N.L.P (1)

- Example

$$\min\ (x_1 - 1)^2 + x_2^{\ 2} - 2$$
$$s.t.\quad x_2 - x_1 = 1$$
$$x_1 + x_2 \le 2$$

Constrained N.L.P (2)

- In Graph

$$\min \; f(x)$$
$$s.t. \quad h(x) = 0$$

Summary of N.L.P. (Fund.)

- Types of N.L.P
  - Unconstraint type
    - Golden section
    - Steepest descent case
  - Constraint type
    - Constraint type → unconstraint type
- Other ways
  - It will be covered in higher grade.