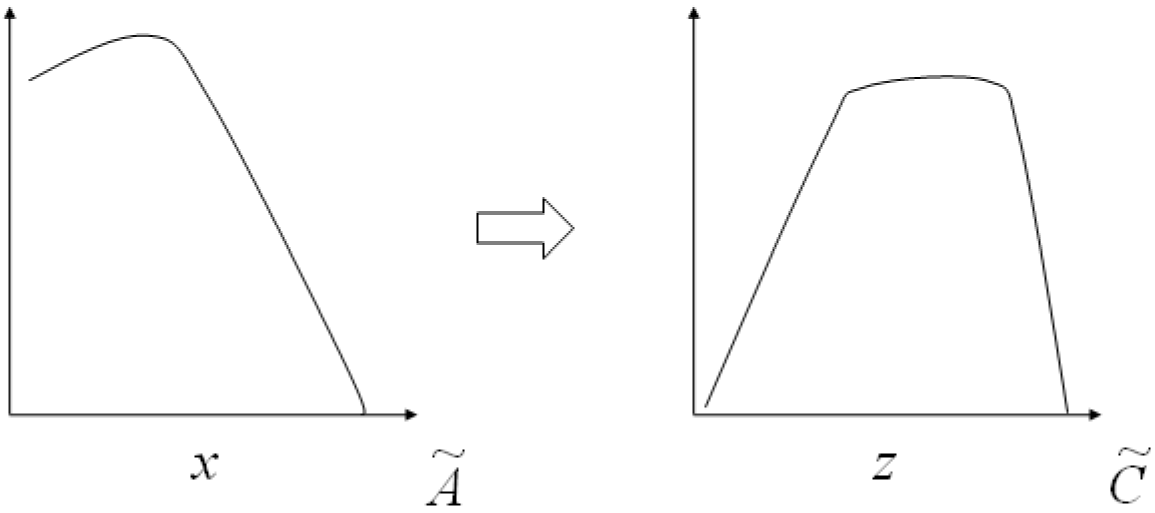
IG0029 IE Computing and Application

Fuzzy Relation, Inference and Control



HYUNSOO LEE

Fuzzy Relation and Inference (1)

- Relation
  - Two set : X and Y
  - Relation : $R_1 : X \geq Y$

R1	1	2	3	4
1				
2				
3				
4				

Fuzzy Relation and Inference (2)

- Two sets' relationship

Fuzzy Relation and Inference (3)

- Another Relation : X < Y

$$(x, y) \in R_1 \equiv \{(1,1),(1,2),(1,3),\cdots,(4,4)\}$$

$$(x, y) \in R_2 \equiv \{(2,1),(3,2),(3,2),(4,1),\cdots,(4,3)\}$$

$$U \times U = R_1 \bigcup R_2$$

$$R_1 \bigcap R_2$$

Fuzzy Relation and Inference (4)

- Another Relation : X = Y

R1	1	2	3	4
1				
2				
3				
4				

$$R_3 \subset R_1$$

Fuzzy Relationship (1)

- Fuzzy Relationship

$$\tilde{R}_1 = X << Y$$

R1	1	2	3	4
1				
2				
3				
4				

$$\tilde{R}_1 \equiv \{(x, y) / \mu_{\tilde{R}(x,y)}\}$$

Fuzzy Relationship (2)

- Another fuzzy relationship : $\tilde{R}_2 = X >> Y$

R1	1	2	3	4
1				
2				
3				
4				

$$\tilde{R}_1 \cup \tilde{R}_2 =$$

$$\tilde{R}_1 \cap \tilde{R}_2 \neq \phi$$

Fuzzy Relationship (3)

- Another fuzzy relationship $\tilde{R}_3 = X \cong Y$

R1	1	2	3	4
1				
2				
3				
4				

$$\tilde{R}_3 \neq (\tilde{R}_1 \cup \tilde{R}_2)^c$$

Fuzzy Inference using fuzzy relation (1)

- Example : "Slender"

"Slender"		Weight (kg)			
		60	65	70	75
Height (cm)	165				
	170				
	175				
	180				

$$\tilde{R}_{\text{"}slender\text{"}} \equiv \{(h,w)/\mu_{slender(h,w)}\}$$

- What is the weight of slender person whose height is 170cm?
- What is the weight of slender person whose height is approximately 170cm?

$$\tilde{A}_{\text{"}approximately\text{"}} \equiv \{165/0.7, 170/1, 175/0.7, 180/0.3\}$$

Fuzzy Inference using fuzzy relation (2)

- Definition of "approximately 170cm"

$$\tilde{A}_{'approximately''} \equiv \{165/0.7, 170/1, 175/0.7, 180/0.3\}$$

"Slender"		Weight (kg)			
		60	65	70	75
Height (cm)	165	1	0.7	0.3	0
	170	1	1	0.7	0.3
	175	1	1	1	0.7
	180	1	1	1	1

Fuzzy Inference using fuzzy relation (3)

- Solving through "Fuzzy Composition"

$$= \begin{bmatrix} 0.7 & 1 & 0.7 & 0.3 \end{bmatrix} \circ \begin{bmatrix} 1 & 0.7 & 0.3 & 0 \\ 1 & 1 & 0.7 & 0.3 \\ 1 & 1 & 1 & 0.7 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

  – Product → Min
  – Sum → Max

Fuzzy Inference using fuzzy relation (4)

- Solving through "Fuzzy Composition"

$$= [0.7 \quad 1 \quad 0.7 \quad 0.3] \circ \begin{bmatrix} 1 & 0.7 & 0.3 & 0 \\ 1 & 1 & 0.7 & 0.3 \\ 1 & 1 & 1 & 0.7 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= [1 \quad 1 \quad 0.7 \quad 0.7]$$

  - What is the weight of slender person whose height is approximately 170cm?

$$\tilde{A}_{\text{"weight for approximately 170"}} \equiv \{60/1, 65/1, 70/0.7, 75/0.7\}$$

Compositional Fuzzy Rule

- " O " ➔ "Max-min"
  - Substitute (+ , X) in matrix algebra

- We can make more complicate inferences & Rules
  - "If height ~ , then weight is ~ "
  - In this case, the weight for the particular height is ?

Another fuzzy inference - Fuzzy number (1)

- Example :

$$\tilde{2} = \{0.5/1,\ 1/2,\ 0.5/3\}$$

$$\tilde{4} = \{0.5/3,\ 1/4,\ 0.5/5\}$$

$$\Downarrow$$

$$\tilde{2} + \tilde{4} = ?$$

Another fuzzy inference - Fuzzy number (2)

- Rule → "Max - Min" Rule

$$\tilde{2} = \{0.5/1,\ 1/2,\ 0.5/3\}$$

$$\tilde{4} = \{0.5/3,\ 1/4,\ 0.5/5\}$$

Fuzzy Inference

- Two types of fuzzy inference
  - 1) Fuzzy relationship → new fuzzy membership function
  - 2) fuzzy numbers' arithmetic operation

Fuzzy control

- Two types of fuzzy controls
  - Mandami model $if\ \ x\ \ is\ \ \tilde{A},\ then\ y\ is\ \tilde{B}$
    - Example:
  - Larsen
  - Tsukamoto
  - Takagi-Sugeno-Kang model (TSK model)
    - Definition : $if\ \ x\ \ is\ \ \tilde{A},\ then\ y\ is\ f(x)$
  - Neural Fuzzy model

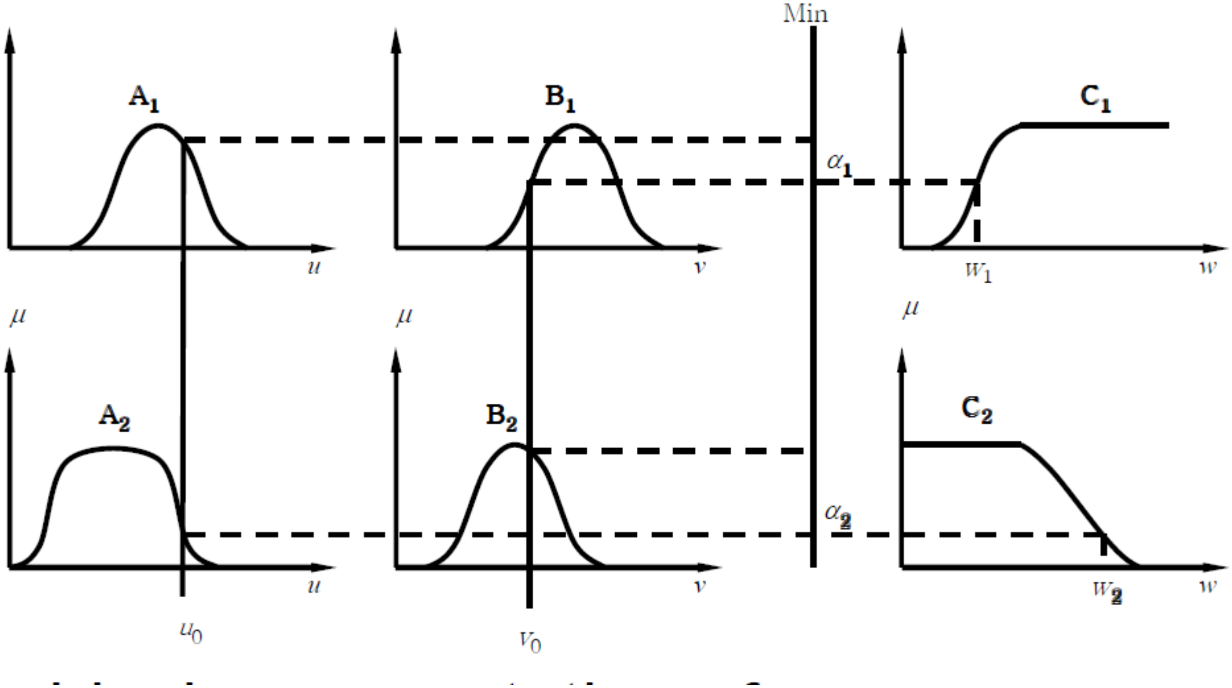
Mandami model

- Example

Tsukamoto model

- 2 or more fuzzy logics
  - Final output → Weighted Sum

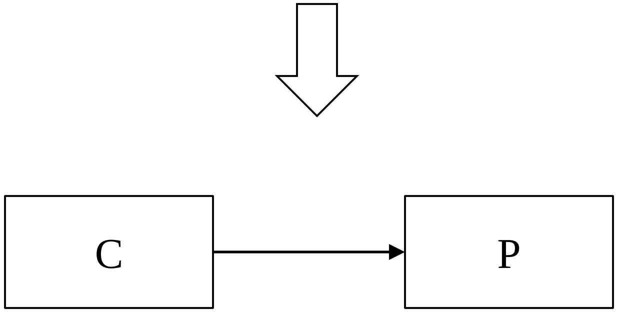
$$w = \frac{\alpha_1 w_1 + \alpha_2 w_2}{\alpha_1 + \alpha_2}$$



TSK model (1)

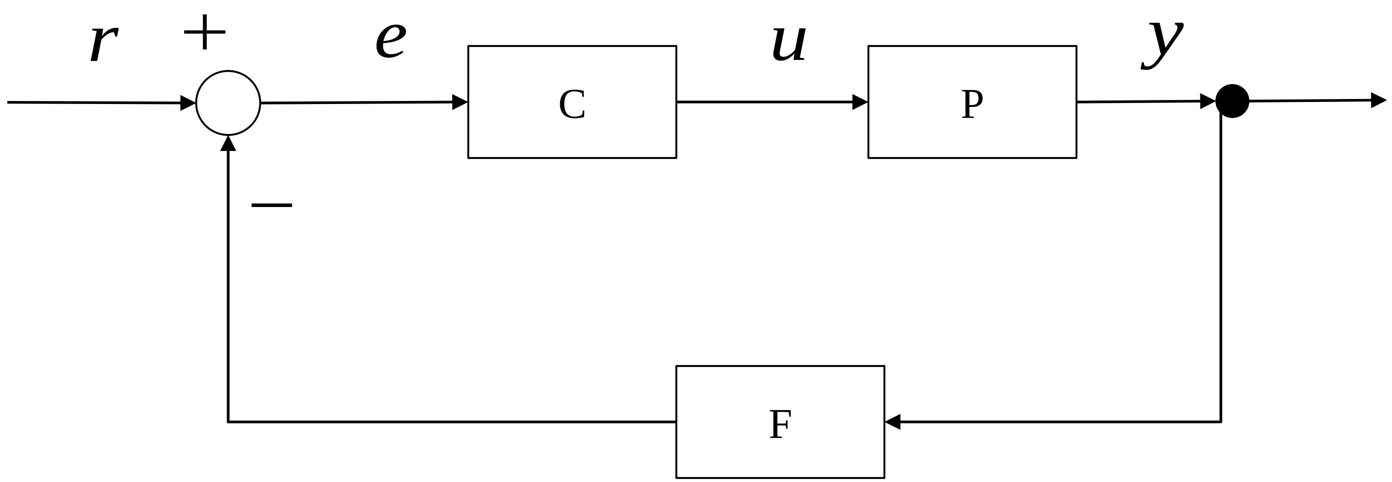
- Control theory

$$y = f(x)$$



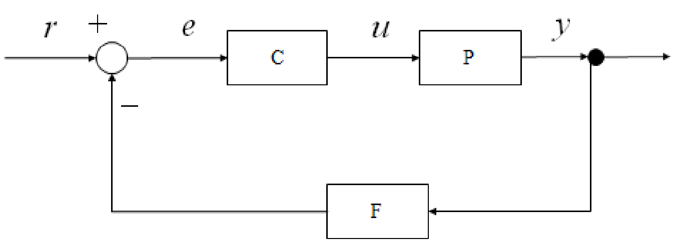
TSK model (2)

- Closed-loop transfer function



TSK model (3)

- Interpretation (1)



$$Y(s) = P(s) \cdot U(s)$$

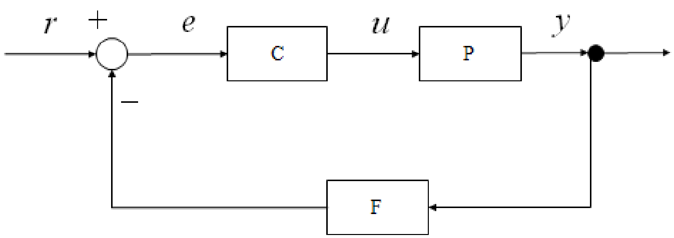
$$U(s) = C(s) \cdot E(s)$$

$$E(s) = R(s) - F(s) \cdot Y(s)$$

$$Y(S) = \left(\frac{P(s) \cdot C(s)}{1 + F(s) \cdot P(s) \cdot C(s)} \right) R(s)$$

TSK model (4)

- Interpretation (2)
  - Closed-loop transfer function

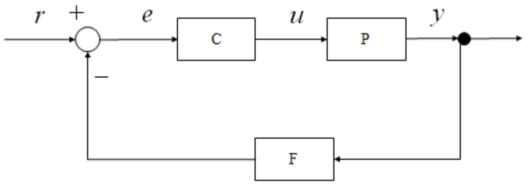


$$Y(S) = \left(\frac{P(s) \cdot C(s)}{1 + F(s) \cdot P(s) \cdot C(s)} \right)$$

$$Y(S) = H(s)R(s)$$

TSK model (5)

- PID Control



  - Most used control design
  - Proportional-Integral-Differential Control

$$u(t) = K_p e(t) + K_I \int e(t)dt + K_I \frac{d}{dt} e(t)$$

  - In case of "MIMO" system

$$u(t) = K_p e(t) + K_I \frac{1}{s} e(t) + K_I t e(t)$$

$$u(t) = \left(K_p + K_I \frac{1}{s} + K_I t \right) e(t)$$

Again, TSK model (1)

- Control model with fuzzy concept

$$if \;\; x \;\; is \;\; \tilde{A}, \; then \; y \; is \; f(x)$$

$$\Downarrow$$

$$if \;\; x_1 \;\; is \;\; M_{i1}, \; and \;\; x_2 \;\; is \;\; M_{i2}, \cdots, \; and \;\; x_n \;\; is \;\; M_{in}$$

$$then$$

$$\dot{x}(t) = A_i x(t) + B_i u(t)$$

Again, TSK model (2)

$$if\ \ x_1\ \ is\ \ M_{i1},\ and\ \ x_2\ \ is\ \ M_{i2}, \cdots, \ and\ \ x_n\ \ is\ \ M_{in}$$

$$then$$

$$\dot{x}(t) = A_i x(t) + B_i u(t)$$

$$\Rightarrow \quad \dot{x}(t) = \sum_{i=1}^{m} \mu_i(t) \{A_i x(t) + B_i u(t)\} / \sum_{i=1}^{m} \mu_i(t)$$

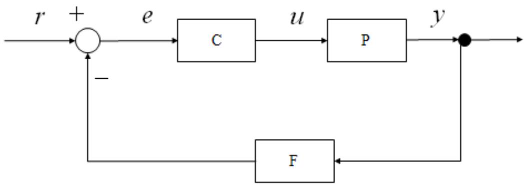
$$\mu_i(t) = \prod_{j=1}^{n} \mu_{M_{ij}}(x_j(t))$$

Source of fuzzy control

- Source of fuzzy control
  - Expert knowledge and control engineering knowledge
  - Observation of operator's action
  - Fuzzy model of the process : linguistic description of the dynamic properties
  - Learning : learning from example of self-organizing learning

Fuzzy control

- Procedure of fuzzy control
  - 0. Situation
  - 1. state variable / Control variable
  - 2. Control model = Mandani / Tsukamoto / TSK / …
  - 3. fuzzification method
  - 4. fuzzy inference
  - 5. defuzzification
  - 6. testing and turning
  - 7. Construction of look-up table



PID Control (1)

- Another control theory →PID Control
  - Most used control design
  - Proportional-Integral-Differential Control

$$U(s) = C(s) \cdot E(s)$$

⇩

$$u(t) = K_p e(t) + K_I \int e(t)dt + K_d \frac{d}{dt} e(t)$$

PID Control (2)

- PID Control

$$u(t) = K_p e(t) + K_I \int e(t)dt + K_d \frac{d}{dt} e(t)$$

$$\Downarrow$$

- Role of "P" $K_p e(t)$: Minimize current error
- Role of "I" $K_I \int e(t)dt$: Reflect past error trends
- Role of "D" $K_d \frac{d}{dt} e(t)$: Prevent future error patterns

PID Control (3)

- Rearrangement of “PID” using Laplace transform

$$u(t) = K_p e(t) + K_I \int e(t)dt + K_d \frac{d}{dt} e(t)$$

Laplace transform

$$u(s) = K_p e(s) + K_I \frac{1}{s} e(s) + K_d s e(s)$$

$$u(s) = \left(K_p + K_I \frac{1}{t} + K_d t \right) e(s)$$

PID Control (4)

- Laplace transform (0)

$$F(s) \triangleright \ell\{ f(t)\} = \int_0^\infty | f(t) | e^{-st} dt$$

$$\ell\{1\} = \int_0^\infty |1| e^{-st} dt = \left[-\frac{1}{s} e^{-st} \right]_0^\infty = \frac{1}{s}$$

PID Control (5)

- Laplace transform (1)

$$F(s) \triangleright \ell\{f(t)\} = \int_0^\infty | f(t) | e^{-st} dt$$

$$\ell\{\frac{df}{dt}\} = \int_0^\infty | f'(t) | e^{-st} dt = [e^{-st} f(t)]_0^\infty + s\int_0^\infty | f(t) | e^{-st} dt$$

$$= -f(0) + s\ell\{f(t)\}$$

$$= sf(s)$$

PID Control (6)

- Laplace transform (2-1)

$$\ell\{\int f(t)dt\} =$$

$$F(s)G(s) = \int_0^\infty e^{-st} f(p)dp \int_0^\infty e^{-sk} g(k)dk$$

$$= \int_0^\infty \int_0^\infty e^{-sp} f(p) e^{-sk} g(k) dp dk$$

$$= \int_0^\infty f(p)dp \int_0^\infty e^{-s(p+k)} g(k)dk$$

$$(t = p + k, dt = dk)$$

PID Control (7)

- Laplace transform (2-2)

$$F(s)G(s) = \int_0^\infty f(p)dp \int_0^\infty e^{-s(p+k)} g(k)dk$$

$$= \int_0^\infty e^{-st} dt \int_0^t f(p) g(t-p)dp$$

$$= \int_0^\infty e^{-st} \left\{ \int_0^t f(p) g(t-p)dp \right\} dt = l\{f * g\}$$

$$l\{\int_0^t f(t)dt\} = \frac{F(s)}{s}$$

PID Control (8)

- Finally,

$$u(t) = K_p e(t) + K_I \int e(t)dt + K_I \frac{d}{dt} e(t)$$

$$u(s) = K_p e(s) + K_I \frac{1}{t} e(s) + K_I s e(s)$$

$$u(s) = \left(K_p + K_I \frac{1}{s} + K_I s \right) e(s)$$

Again, TSK model (1)

- Control model with fuzzy concept

$$if \;\; x \;\; is \;\; \tilde{A}, \; then \; y \; is \; f(x)$$

⇩

$$if \;\; x_1 \;\; is \;\; M_{i1}, \; and \; x_2 \;\; is \;\; M_{i2}, \cdots, \; and \;\; x_n \;\; is \;\; M_{in}$$

$$then$$

$$u(s) = \left(K_p + K_I \frac{1}{s} + K_I s \right) e(s)$$

Again, TSK model (2)

- TSK model

$$if\ \ x_1\ \ is\ \ M_{i1},\ and\ x_2\ \ is\ \ M_{i2},\cdots,\ and\ \ x_n\ \ is\ \ M_{in}$$

$$then$$

$$u(s)=\left(K_p+K_I\frac{1}{s}+K_I s\right)e(s)$$

$$\Downarrow$$

$$if\ \ x_1\ \ is\ \ M_{i1},\ and\ x_2\ \ is\ \ M_{i2},\cdots,\ and\ \ x_n\ \ is\ \ M_{in}$$

$$then$$

$$\dot{x}(t)=A_i x(t)+B_i u(t)$$

Again, TSK model (3)

$$if\ \ x_1\ \ is\ \ M_{i1},\ and\ \ x_2\ \ is\ \ M_{i2}, \cdots, and\ \ x_n\ \ is\ \ M_{in}$$

$$then$$

$$\dot{x}(t) = A_i x(t) + B_i u(t)$$

$$\Rightarrow \quad \dot{x}(t) = \sum_{i=1}^{m} \mu_i(t) \{ A_i x(t) + B_i u(t) \} / \sum_{i=1}^{m} \mu_i(t)$$

$$\mu_i(t) = \prod_{j=1}^{n} \mu_{M_{ij}}(x_j(t))$$

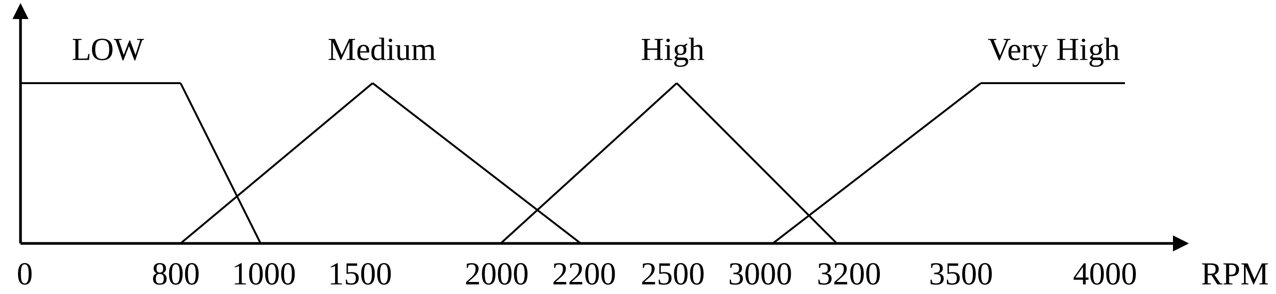
Example (1)

- Engine Speed(rpm) & torque (ft-lb)
  - If rpm is low then torque is low
  - If rpm is medium then torque is medium
  - If rpm is high then torque is high
  - If rpm is very high then torque is medium

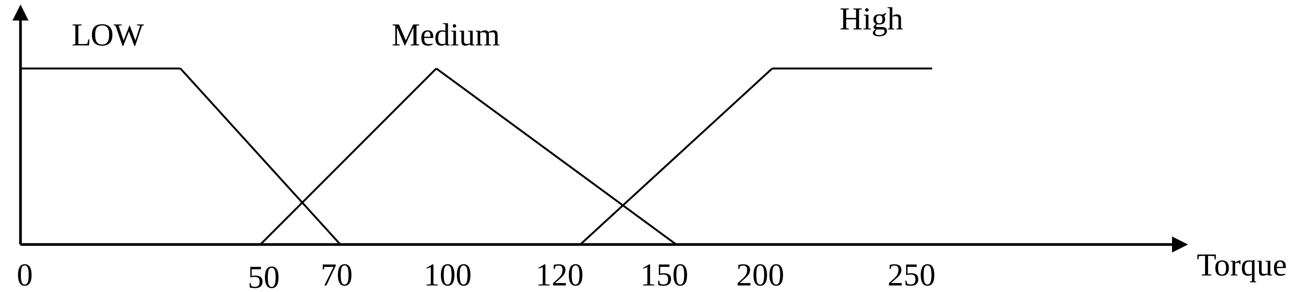
- How to control using TSK model

Example (2)

- Fuzzy membership functions
  - RPM



  - Torque



Example (3)

- Fuzzy graph for fuzzy "Engine-Torque" logics

Example (4)

- TSK control model for fuzzy “Engine-Torque” logics

Example (5)

- Solution for TSK model

Example (5)

- Now, torque = 1100

Source of fuzzy control

- Source of fuzzy control
  - Expert knowledge and control engineering knowledge
  - Observation of operator's action
  - Fuzzy model of the process : linguistic description of the dynamic properties
  - Learning : learning from example of self-organizing learning

Fuzzy control

- Procedure of fuzzy control
  - 0. Situation
  - 1. state variable / Control variable
  - 2. Control model = Mandani / Tsukamoto / TSK / …
  - 3. fuzzification method
  - 4. fuzzy inference
  - 5. defuzzification
  - 6. testing and turning
  - 7. Construction of look-up table

Exercise

- Total
  - 1) Calculate

$$\int_{1}^{2} \tilde{2}x^{3} + \tilde{4}x^{2}dx$$

- Where

$$\tilde{2} = \{0.5/1, 1/2, 0.5/3\}$$
$$\tilde{4} = \{0.5/3, 1/4, 0.5/5\}$$

- 10 points

Exercise

- 2) Suppose that we have two fuzzy relations (R and S)

$$R = \begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.0 & 0.2 & 0.8 \\ 0.3 & 0.6 & 1.0 \end{bmatrix} \end{array} \qquad S = \begin{array}{c} \\ y_1 \\ y_2 \\ y_3 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.3 & 0.7 & 1.0 \\ 0.5 & 1.0 & 0.6 \\ 1.0 & 0.2 & 0.0 \end{bmatrix} \end{array}$$

  - Based on predefined fuzzy relations R&S, make a new look up table for describing relationship between x and z - 20 points]

- 3). Suggest TSK model for your mobile phone – 30 pts
  - Explicitly defined your $y = f(\cdot)$

Exercise

- 4) Consider the problem of correlating interest rate (IR) to unemployment (UE). (Note that this is a simplistic approach and does not capture the realism of the problem). Let us the following fuzzy sets be defined over their respective domains - 40 pts
  - IR
    - Low : $Low = \{1/1, 1/2, 0.5/3\}$
    - Med : $Medium = \{0.5/3, 1/4, 1/5, 1/6, 0.5/7\}$
  - UE
    - Low : $Low = \{1/4, 1/5, 0.3/6\}$
    - Med : $Medium = \{0.5/6, 1/7, 1/8, 0.5/9\}$
  - And, let us define two simple rules of the form
    - If IR is low then UE is low
    - If IR is medium then UE is medium
  - Let another IR= $\{0.7/1, 1/3, 0.7/4, 0.2/5\}$
  - Find UE for this another IR