< June 13th Monday >

Quantum

gate

* Quantum algorithm.,

1. Deutsch algorithm
   (Deutsch-jozsa algorithm.
2. QFT (Quantum Fourier Transformation.
   (phase estimation algorithm.
3. Shor algorithm.
4. Grover algorithm (Quantum Search

$|\psi\rangle = |0\rangle$

state

qubit $\Rightarrow$ one qubit system

$|\psi'\rangle = |00\rangle$

state

two qubits system

$|\psi\rangle = |\psi_{n-1} \cdots \psi_0\rangle$

State

n-qubit system

$|\psi\rangle = |0\rangle \Rightarrow \alpha|0\rangle + \beta|1\rangle$

$|-|$

$H^2 = I$

$$|\psi\rangle \longrightarrow \boxed{H} \longrightarrow |\psi'\rangle$$

Superposition state

$$|\psi\rangle \longrightarrow \boxed{H} \longrightarrow \boxed{H} \longrightarrow |\psi\rangle$$

$$H \cdot H^{\dagger} = I$$

$$|\psi\rangle - \boxed{P(\theta)} - \boxed{H} \rightarrow \boxed{Z} - |\psi'\rangle$$

$$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \qquad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Downarrow$$

$$\boxed{|\psi'\rangle = Z \cdot H \cdot P(\theta) \cdot |\psi\rangle}$$

< Parallel Composition >

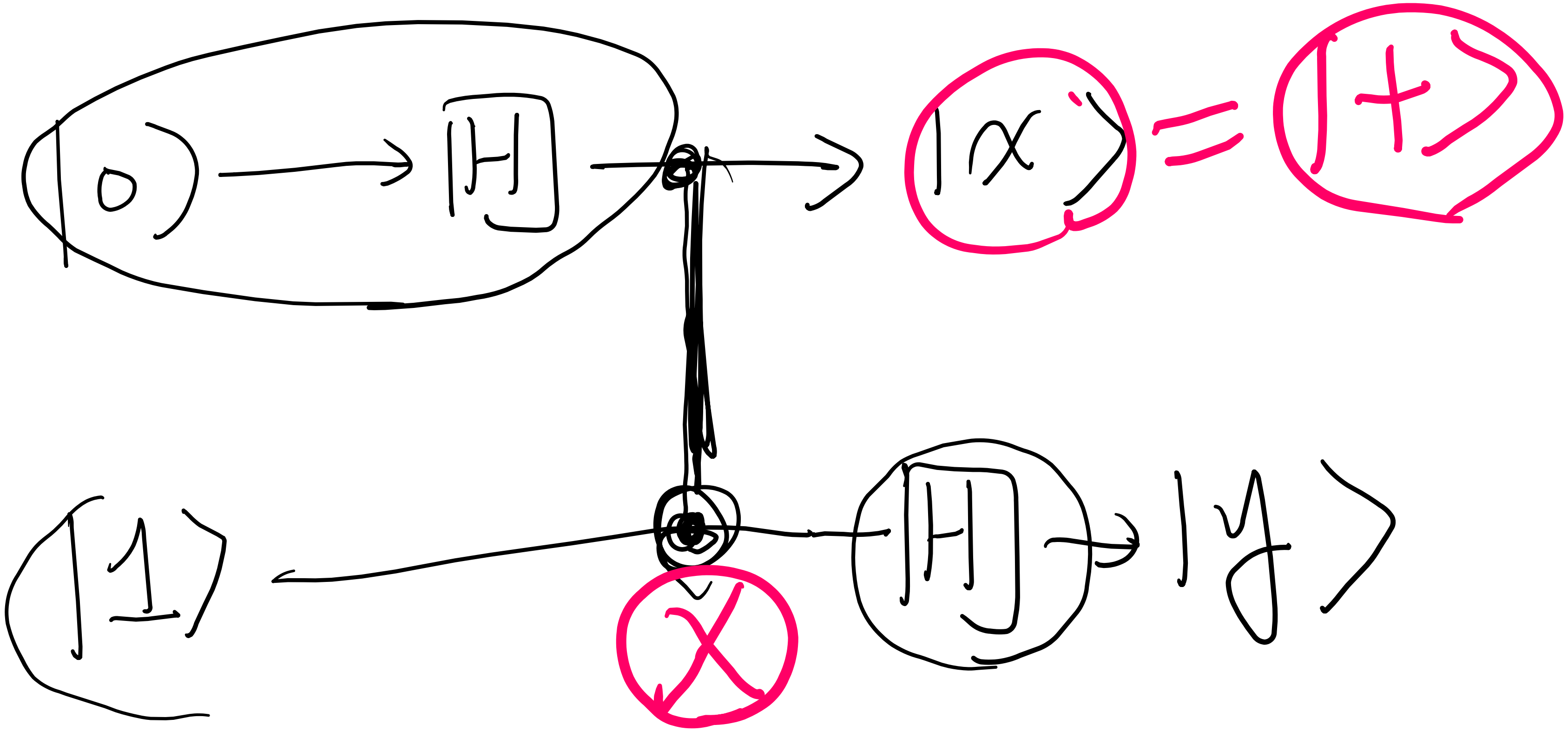
$$|0\rangle \longrightarrow \boxed{H} \longrightarrow |x\rangle$$

$$|0\rangle \longrightarrow \boxed{H} \longrightarrow |y\rangle$$

$$\Downarrow$$

$$|x,y\rangle = H \otimes H \, |0\rangle|0\rangle \Rightarrow |x,y\rangle$$

one state



$$|x\rangle = H \cdot |0\rangle$$

$$|y\rangle = H \cdot (H \cdot |0\rangle) \otimes |1\rangle = H \cdot H$$

* Quantum interference

Let $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle$.

$$\boxed{\boxed{H \cdot |\psi\rangle} = |0\rangle}$$

$|0\rangle \Rightarrow \frac{1}{2} \rightarrow 1$ positive interference

$|1\rangle \Rightarrow \frac{1}{2} \rightarrow 0$ negative inter

<Deutsch's algorithm>

Quantum algorithm

① **Objective**

$$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } x = 1 \end{cases}$$

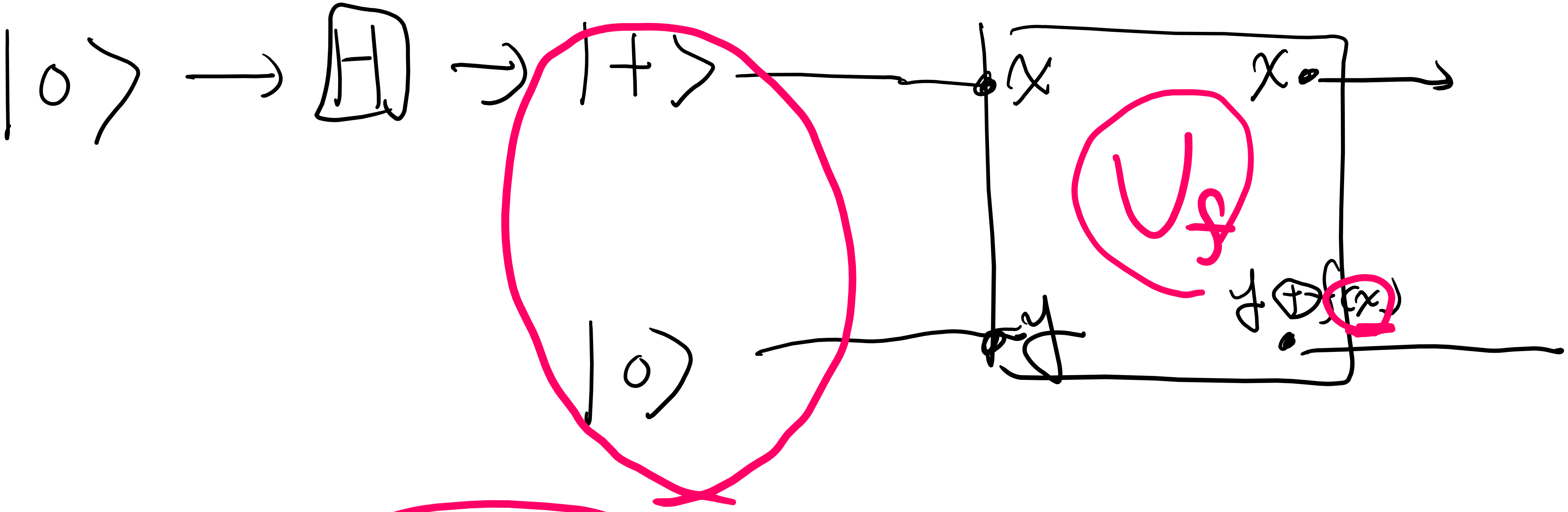
balance function

U_f

XOR

$$\boxed{U_f\,|x, y\rangle \rightsquigarrow |x, y \oplus f(x)\rangle}$$

Definition.



$$U_f |+0\rangle = |+ , 0 \oplus f(+)|$$

$$U_f\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|0\rangle = \frac{1}{\sqrt{2}}\left(U_f|0\rangle|0\rangle + U_f|1\rangle|0\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|0, 0\oplus \underline{f(0)}\rangle + \frac{1}{\sqrt{2}}|1, 0\oplus\right.$$

if is

balance

fnuth.

$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$$U_f\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|0\rangle = \frac{|0, f(0)\rangle + |1, f(1)\rangle}{\sqrt{2}}$$

$$\boxed{|\psi_{out}\rangle = (H \otimes I) U_f (H \otimes H)|0\rangle|1\rangle}$$

$|+-\rangle$

$\Downarrow$

$$\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$$

$$U_f|00\rangle = |0, 0 \oplus f(0)\rangle = |0, f(0)\rangle$$

$$= (1 - f(0))|00\rangle + f(0)|01\rangle$$

$$\begin{cases} |\psi_{out}\rangle = -|0\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \\ |\psi_{out}\rangle = \pm|1\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \end{cases}$$

$\boxed{\text{if } f(0) = f(1)}$ → f is constant

if $f(0) \neq f(1)$

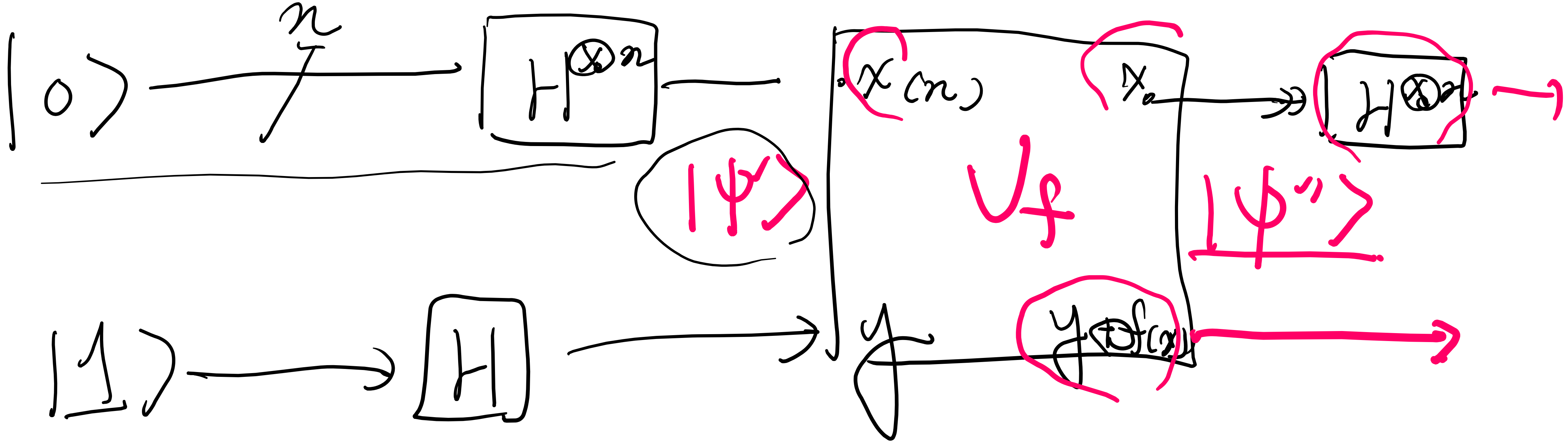
* Extend Deutsch's algorithm

" Deutsch-Jozsa algorithm.

using a specialized gate & input

observe ' output '

balance or constant

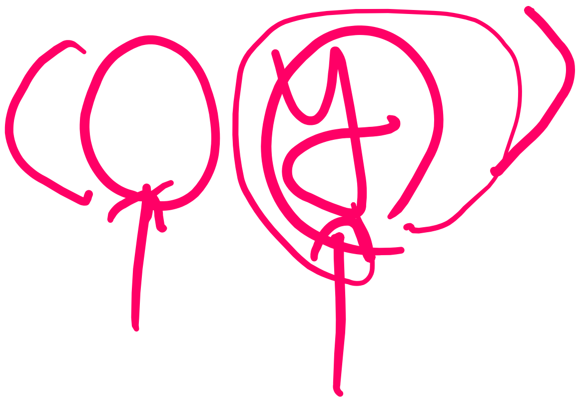


$$\boxed{|\psi'\rangle = \left(H^{\otimes n} \cdot |0\rangle^{\otimes n}\right) \otimes \left(H \cdot |1\rangle\right)}$$

$$|\psi'\rangle = \left(\frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) \right)$$

$$|\psi''\rangle = \frac{1}{\sqrt{2^n}} \sum_x (-1)^{f(x)} |x\rangle \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right)$$

$$\Downarrow$$



$|0\rangle \Longrightarrow$ Constant $f(x)$

$|1\rangle \Longrightarrow$ balance function.

Example $\boxed{f(x) = 1}$ $\Rightarrow$ constant function.

$$|\psi_{out}\rangle = |0\,\underset{\substack{\shortparallel \\ 0}}{0}\rangle$$

Example

$$\begin{cases} ~~f(00)=f(01)=0~~ \\ ~~f(10)=f(11)=1~~ \end{cases} \text{~~balance~~}$$

$$|\phi_{out}\rangle = |10\rangle\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$$

1

2. Quantum Fourier Transformation (QFT)

$$|x\rangle = |\underbrace{x_{n-1} \cdots x_0}_{n}\rangle$$

$$|y\rangle = |y_{n-1} \cdots y_0\rangle$$

Def.

$$\boxed{x \cdot y = x_0 \cdot y_0 + \cdots + x_{n-1} \cdot y_{n-1}}$$

$$|\psi\rangle = |x_{n-1}\, x_{n-2} \cdots x_0\rangle$$

$$\Big\Downarrow \text{Q.F.T} \quad U_F$$

$$\frac{1}{\sqrt{2^{\frac{n}{2}}}}\left(|0\rangle + e^{2\pi i[0.x_{n-1}]}|1\rangle\right) \otimes \left(|0\rangle + e^{2\pi i[0.x_{n-2}x_{n-1}]}|1\rangle\right) \cdots \otimes \left(|0\rangle + e^{2\pi i[0.x_0 x_1 \cdots x_{n-1}]}|1\rangle\right)$$

$$0.x_0 = \frac{x_0}{2}$$

$$0.x_0 x_1 = \frac{x_0}{2^2} + \frac{x_1}{2}$$

$$0.x_0 x_1 x_2 = \frac{x_0}{2^3} + \frac{x_1}{2^2} + \cdots$$

$|\psi\rangle \xrightarrow{QFT}$ Some combination of

$$(|0\rangle + e^{i\square}$$

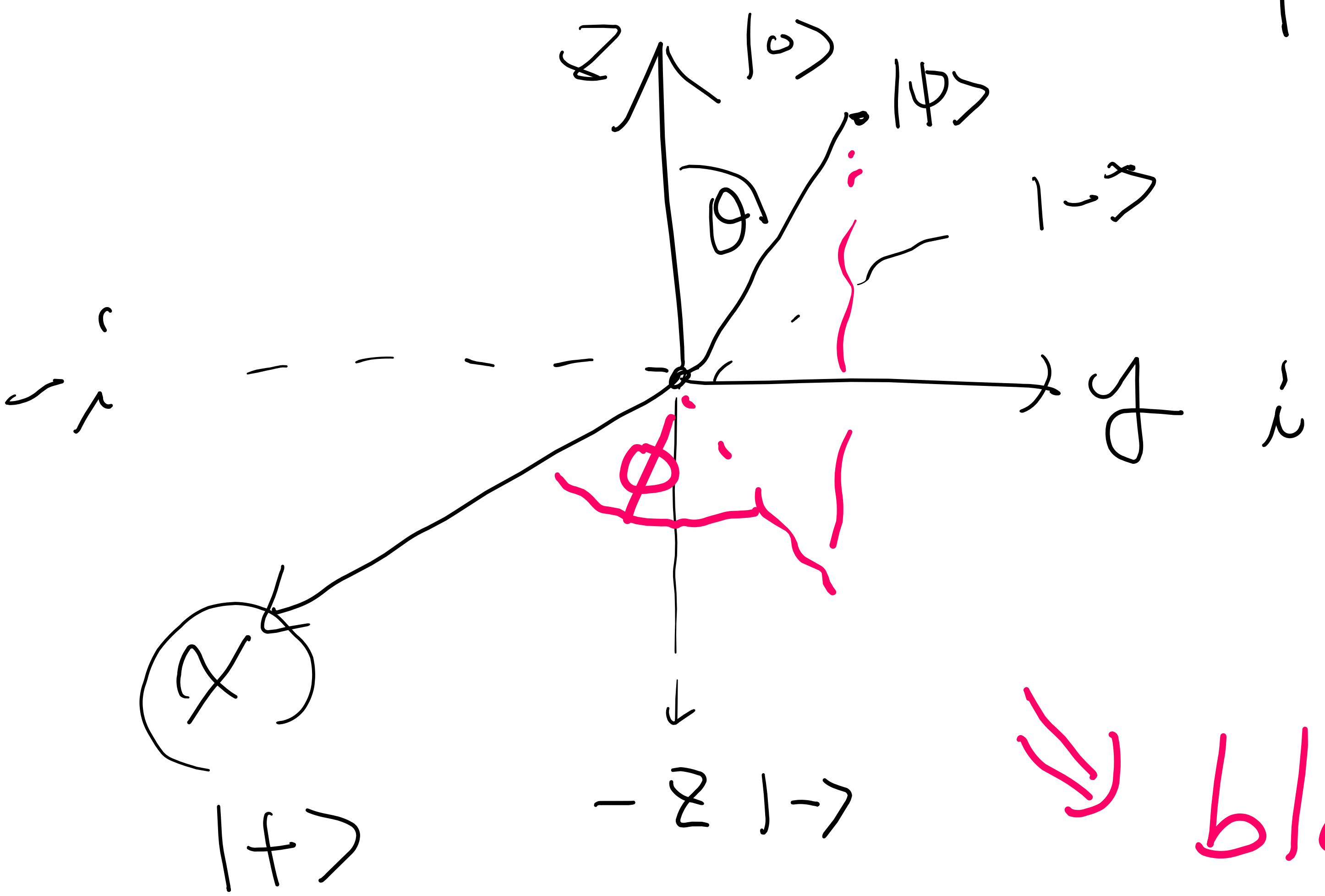
$\otimes \cdots$

phase

$\Downarrow$

Superpositin

phase transformatn.



$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}$$

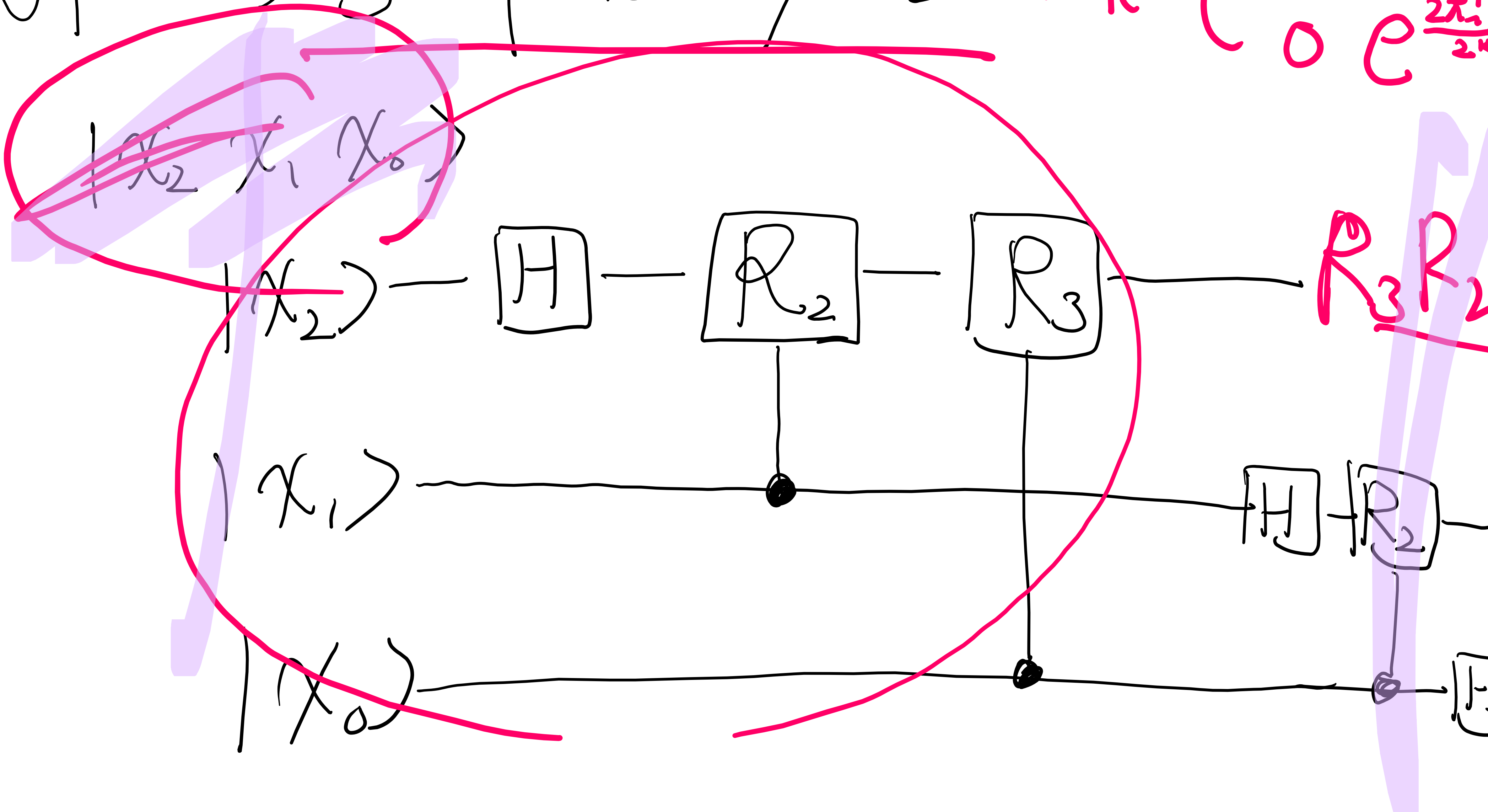
phase angle

⇒ bloch spere

$$\underbrace{U_F}_{QFA} \cdot |x\rangle = \frac{1}{\sqrt{2^n}} \cdots \cdots$$

$U_F \sim$ 3-qubit system

$R_k = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{2\pi i}{2^k}} \end{pmatrix}$



* QFT is need for "Rotation angle"

phase angle

with "Z" axis

phase operator ⇒ U

$$U|\phi_n\rangle = e^{2\pi i \cdot \theta_n}|\phi_n\rangle$$

phase operation.

using Q.F.T

"we can check "each phase angle" in each qubit"

QFT gate

* Shor's algorithm.

"$x^{r} = 1 \pmod{N}$"

The Smallest r

$x^r = 1 \pmod N$

$\Downarrow$

$\begin{cases} x = 5 \\ N = 44 \end{cases}$

5

$r = 5$

$5^5 = 3125 = 71 \times 4$

$5^r = 1 \pmod{44}$

$r = 3 \Rightarrow 5^3 = 125 = 44 \times 2 + 37$

$\Downarrow$

$37 \neq 1$

$r = 4 \Rightarrow 5^4 = 625 = 44 \times 14 + 9 \Rightarrow 9 \neq$

$x^r = \underline{1} \pmod{N}$

Shor's algorithm

if $N \to \infty$, what is r?

operator $U_x |y\rangle = \begin{cases} |xy \bmod N\rangle & 0 \le y \le N-1 \\ |y\rangle & N \le y \le 2^{L} \end{cases}$

$\boxed{U_{x}|v_t\rangle = e^{2\pi i (\frac{s}{r})} |v_t\rangle}$

phase angle

Spectrum of U_x

eigen vector of U_x

log

inf

$$\left|\frac{1}{r}\right\rangle = \frac{1}{\sqrt{s}} \sum_{k=0}^{s-1} e^{-\frac{2\pi i k}{r}} |k\rangle$$

qubit

Shor's algorithm.

4. Grover's algorithm (Quantum Search.)

5 4 6 2 1 7 8 9 . . . .

Smallest one

Sorted order (Ascending
decending.)