< May 30th >

↳ Quantum gate

$\Downarrow$

operator

$\Downarrow$

unitary operator

$$U = U^{-1} = U^{\dagger}$$

$$U^2 = U \cdot \underbrace{U^{\dagger}}_{\text{adjoint}} = I$$

$|0\rangle \longrightarrow \boxed{X} \longrightarrow |1\rangle$

basig qubit

$|0\rangle \xrightarrow{?} |1\rangle$

$\boxed{X|0\rangle \Rightarrow |1\rangle}$

Then, I will classical gate

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Quantum gate. universality

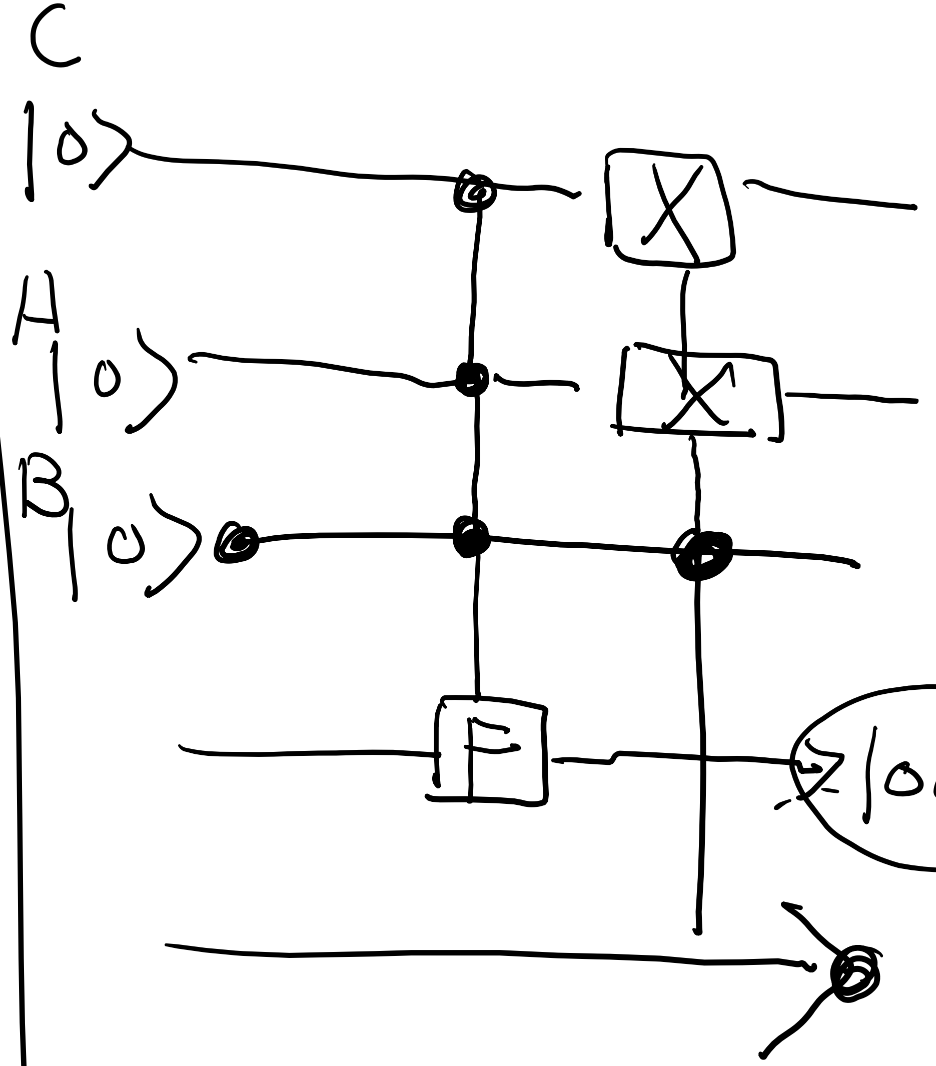
Nand gate

A	B	A Nand B (Not and)
0	0	→ 0 ⇒ 1
0	1	→ 0 ⇒ 1
1	0	→ 0 ⇒ 1
1	1	→ 1 ⇒ 0

1/A Fredkin gate (Universality)

Control bit

$\boxed{C}$	A	B	A'	B'
0	0	0	0	0
0	0	1	0	1
0	1	0	1	0
0	1	1	1	1
1	0	0	0	0
1	0	1	1	0
1	1	0	0	1
1	1	1	1	1



(Toffoli gate)

XOR

C_1	C_2	C_1 & C_2 (AND)	T	T'
0	0	0	0	0
0	0	0	1	1
0	1	0	0	0
0	1	0	1	1
1	0	0	0	0
1	0	0	1	1
1	1	1	0	1
1	1	1	1	0

XOR

00 $\Rightarrow$ 0

01 $\Rightarrow$ 1

10 $\Rightarrow$ 1

11 $\rightarrow$ 0

$\oplus$

* trasit to "Quantum gate."

$$U_{Not} = X = \begin{pmatrix} \boxed{0} & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \langle 0|X|0\rangle & \langle 0|X|1\rangle \\ \langle 1|X|0\rangle & \langle 1|X|1\rangle \end{pmatrix}$$

Dirac representation

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\Rightarrow a|0\rangle\langle 0| + b|0\rangle\langle 1| + c|1\rangle\langle 0| + d$$

$$|1\rangle\langle 1|$$

$$
\begin{array}{c|cccc}
 & 00 & 01 & 10 & 11 \\
\hline
00 & & & & \\
01 & & & & \\
10 & & & & \\
11 & & & &
\end{array}
\quad
\begin{array}{c c}
 & \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\
\begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array} & \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}
\end{array}
\Rightarrow
\begin{aligned}
&1 \cdot |00\rangle\langle 00| \\
&+ 2 \cdot |00\rangle\langle 01| \\
&+ 3 |00\rangle\langle 10| \\
&+ 4 |00\rangle\langle 11| \\
&+ \dots
\end{aligned}
$$

$$|ab\rangle\langle cd| = \underline{\langle a|c\rangle\langle b|d\rangle}$$

* Mixing gate.

$$(H \otimes H) = U_{Not}^{\otimes} \quad = \quad \boxed{U_{Not}^{H}}$$

$$|+\rangle \longrightarrow |-\rangle$$

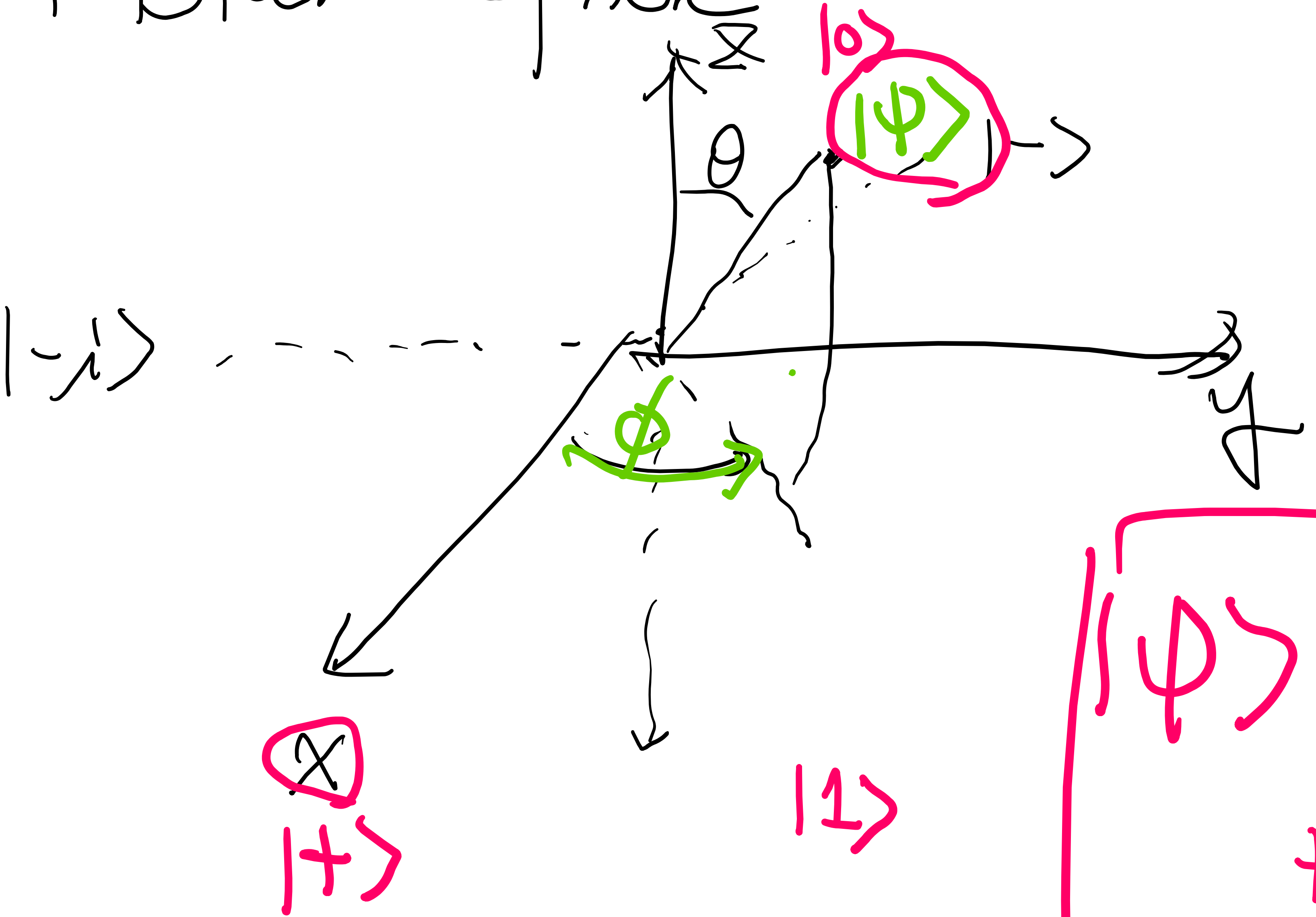
$$\boxed{|+\rangle \to |0\rangle \;\cancel{\to}\; |1\rangle \to |-\rangle}$$

$$U_{NOT}^{H} = H \, U_{Not} \cdot H =$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= Z$$

* Bloch Sphere



$$0 \le \theta \le \pi$$
$$0 \le \phi \le 2\pi$$

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

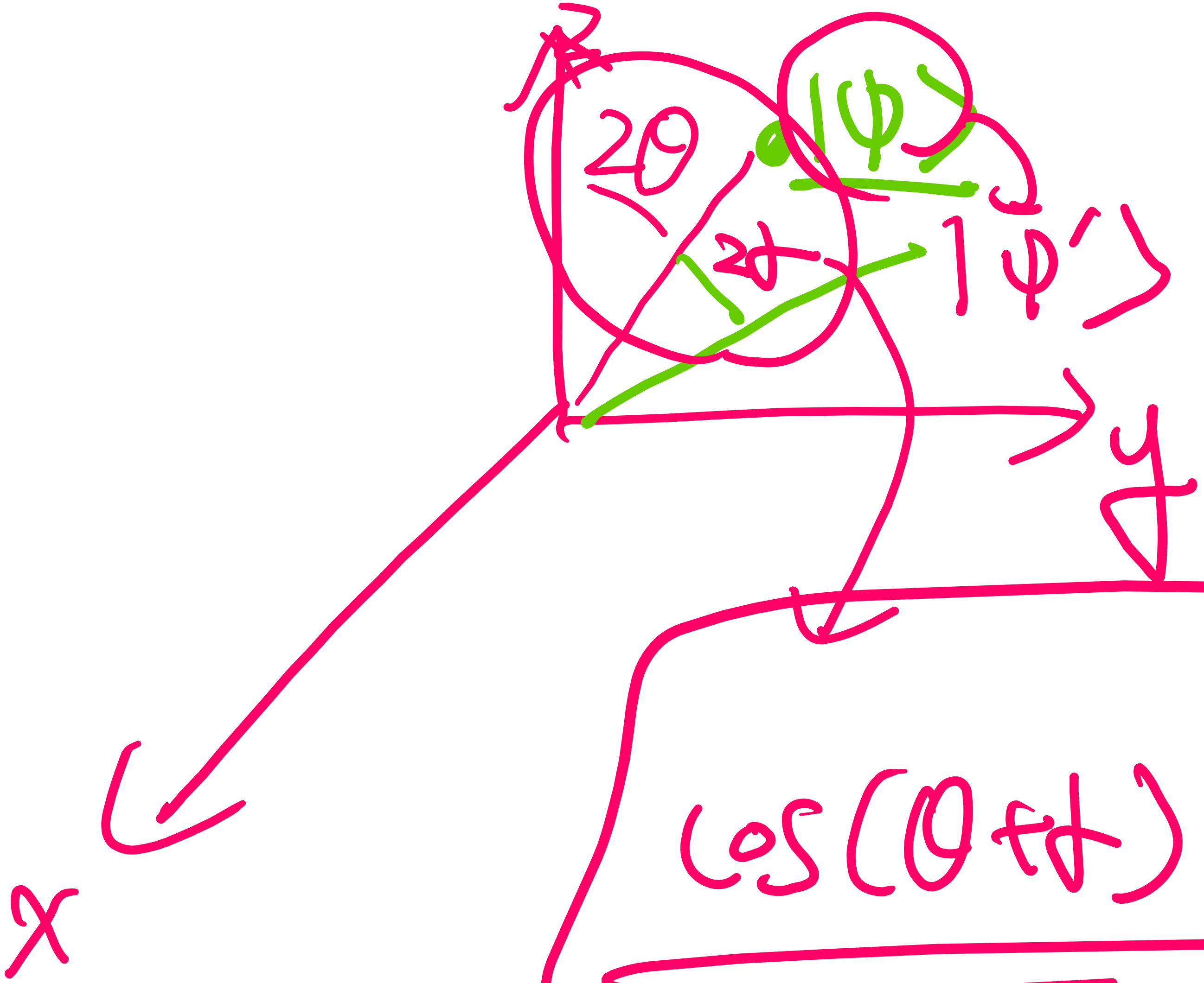
$|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \rightarrow \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$

$R(\phi)$

Rotation angle

$\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \leftarrow R(\phi) \cdot |\psi\rangle$

$= \begin{pmatrix} \cos\phi\cos\theta - \sin\phi\sin\theta \\ \sin\phi\cos\theta + \cos\phi \cdot \sin\theta \end{pmatrix} = \begin{pmatrix} \cos(\phi+\theta) \\ \sin(\phi+\theta) \end{pmatrix}$



$$\cos(\theta+\alpha)\,|0\rangle + \sin(\theta+\alpha)\,|1$$

phase flip gate $\Rightarrow Z$.

phase shift gate $\Rightarrow Y$ or P

$$P = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$|0\rangle\langle 0| + e^{i\theta}|1\rangle\langle 1|$$

$$P \cdot |\phi\rangle = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} \alpha \\ \beta e^{i\theta} \end{pmatrix}} \Rightarrow \alpha|0\rangle + e^{i\theta} \cdot \beta|1\rangle$$

$$\boxed{\theta = \pi}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Z gate

$$\Rightarrow \boxed{\alpha|0\rangle - \beta|1\rangle}$$

$$|\varphi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

phase factor

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Z

X rotation.

* S-gate $\Rightarrow$ $\boxed{\theta = \frac{\pi}{2}, \phi = 0}$

t-gate $\Rightarrow$ $\theta = \frac{\pi}{4}, \phi = 0$

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$$e^{i\frac{\pi}{2}} = \boxed{\cos\frac{\pi}{2}} + i \boxed{\sin\frac{\pi}{2}}$$

$$\phantom{e^{i\frac{\pi}{2}}} \quad 0 \qquad\quad 1$$

$$= i$$

* Expansion of "$\underline{\underline{U}}$" unitary gate

$$U \cdot U^{\dagger} = U \cdot U^{-1} = U \cdot U = I$$

$$\boxed{U^2 = I}$$

$$\boxed{e^{-i \cdot \theta \cdot U}} \Rightarrow \text{operator}$$

$$e^{-i\cdot\theta\cdot U} = I - i\cdot\theta\cdot U + \frac{(-i)^2}{2!}\theta^2 U^2 + (-i)^3\frac{\theta^3}{3!}U^3$$

$$= \cos\theta\cdot I - i\sin\theta\cdot U$$

$$+ \frac{(-i)^4}{4!}\theta^4 U^4 + (-i)^5\frac{\theta^5}{5!}U^5 + \cdots$$

$$= \left(I - \frac{\theta^2}{2}I + \frac{\theta^4}{4!}I + \cdots\right) - i\theta U + i\frac{\theta^3}{3!}U - i\frac{\theta^5}{5!}U + \cdots$$

$$\left(1 - \frac{\theta}{2!} + \frac{\theta^4}{4!} - \cdots\right)I - i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots\right)U$$

$$\cos\theta \qquad\qquad \sin\theta$$

$$U = e^{ia} \cdot R_x(b) \cdot R_y(c) \cdot R_z(d)$$

* extent 2 qubit gate

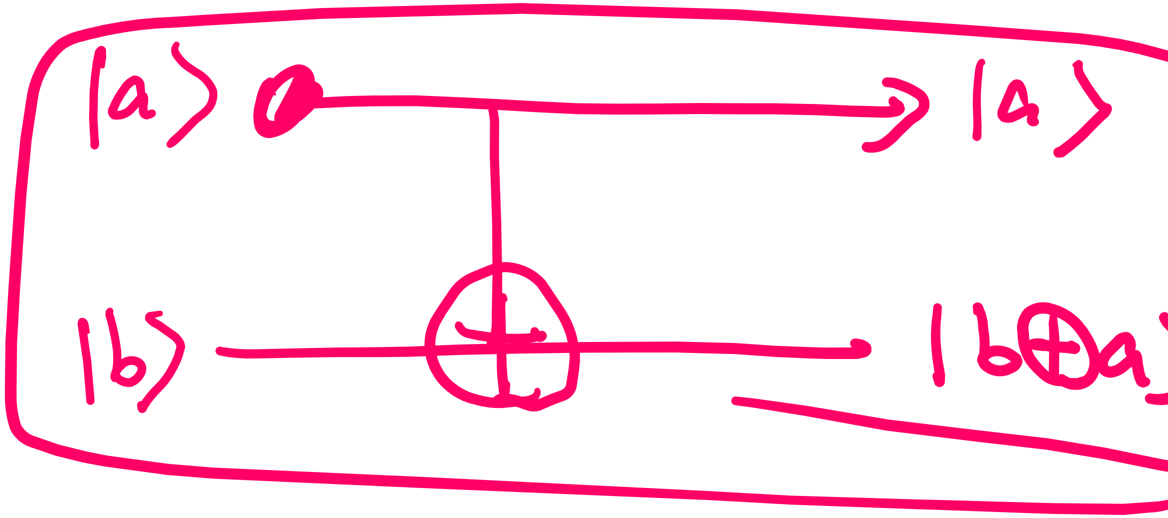
$|ab\rangle$

first famous Qubit gate $\Rightarrow$ CNot gate

CN

$|ab\rangle \xrightarrow{CN} |a, b \oplus a\rangle$

$$CN = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



$$\Rightarrow \boxed{CN = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 11| + |11\rangle\langle 10|}$$

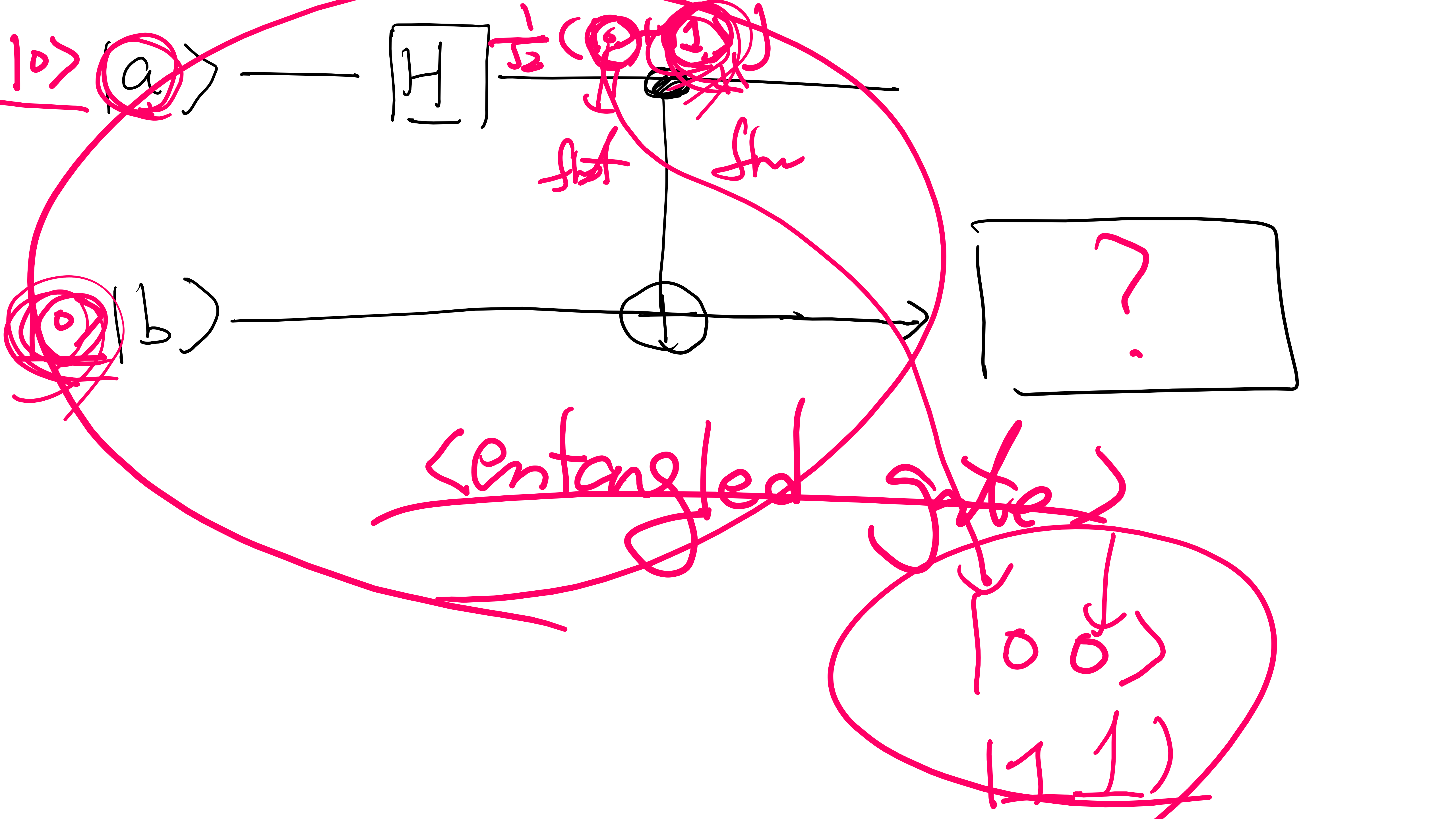
$$CN \cdot |10\rangle$$

$$= \left(|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 11| + |11\rangle\langle 10|\right) |10\rangle$$

$$= |11\rangle$$

$$|\psi\rangle = \alpha|10\rangle + \beta|11\rangle$$

$$CN \cdot |\psi\rangle \Rightarrow \alpha|11\rangle + \beta|10\rangle$$



|0⟩
|a⟩
H
1/√2 (0 + 1)
first
then
0
|b⟩
?
<entangled state>
|00⟩
|11⟩

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}} \Rightarrow |\beta_{00}\rangle$$

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}} \Rightarrow |\beta_{01}\rangle$$

$$\frac{|00\rangle - |11\rangle}{\sqrt{2}} \Rightarrow |\beta_{10}\rangle$$

$$\frac{|01\rangle - |10\rangle}{\sqrt{2}} \Rightarrow |\beta_{11}\rangle$$

$$CN\left(|00\rangle + |10\rangle\right)$$

$$= \left(|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 11| + |11\rangle\langle 10|\right)\left(|00\rangle + |10\rangle\right)$$

$$= \boxed{|00\rangle + |11\rangle}$$

* Second 2 qubit gate

<u>CH gate</u> $= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

$CH|01\rangle = |01\rangle$

$CH|11\rangle = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$