⟨May 16th Monday⟩

Quantum entanglement

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle$$

!! Not entangle!

$$|01\rangle \Rightarrow |0\rangle \otimes |1\rangle$$

$$\boxed{|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle}$$

$$\boxed{|\psi'\rangle = a'|01\rangle + b'|10\rangle}$$

$$|\psi\rangle = \frac{1}{4}|0\rangle + \frac{1}{5}|1\rangle$$

$$|\psi'\rangle = \frac{1}{5}|00\rangle + \frac{1}{6}|11\rangle + \frac{2}{3}|01\rangle$$

$$|\psi\rangle = \boxed{\frac{1}{4}|001\rangle + \frac{1}{6}|110\rangle}$$

$$\boxed{|01?\rangle}$$

?

< Complex System >

$$|\psi\rangle = \frac{1}{\sqrt{8}}|00\rangle + \sqrt{\frac{3}{8}}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$$

① Prob. of $|01\rangle \Rightarrow \frac{3}{8}$

$\cancel{\langle \phi | \psi \rangle}$

$\langle 01 | \psi \rangle$?

② prob $|0\rangle$

density $\rho = |0\rangle\langle 0|$

density

$\langle \psi | \rho | \psi \rangle$

$\frac{1}{\sqrt{8}}|00\rangle + \frac{\sqrt{3}}{8}|01\rangle$

$\frac{1}{2}$

* Bell's inEquality

basis #1 basis #2

$$\rho = \frac{1}{3}|u_1\rangle\langle u_1| - i\frac{\sqrt{2}}{3}|u_1\rangle\langle u_2| + i\frac{\sqrt{2}}{3}|u_2\rangle\langle u_1| + \frac{2}{3}|u_2\rangle$$

$$\text{Prob. of } |u_2\rangle \Rightarrow \frac{2}{3}$$

$$\boxed{P = \mathrm{Tr}\left(|u_2\rangle\langle u_2| \cdot \rho\right)}$$

quantum entanglement

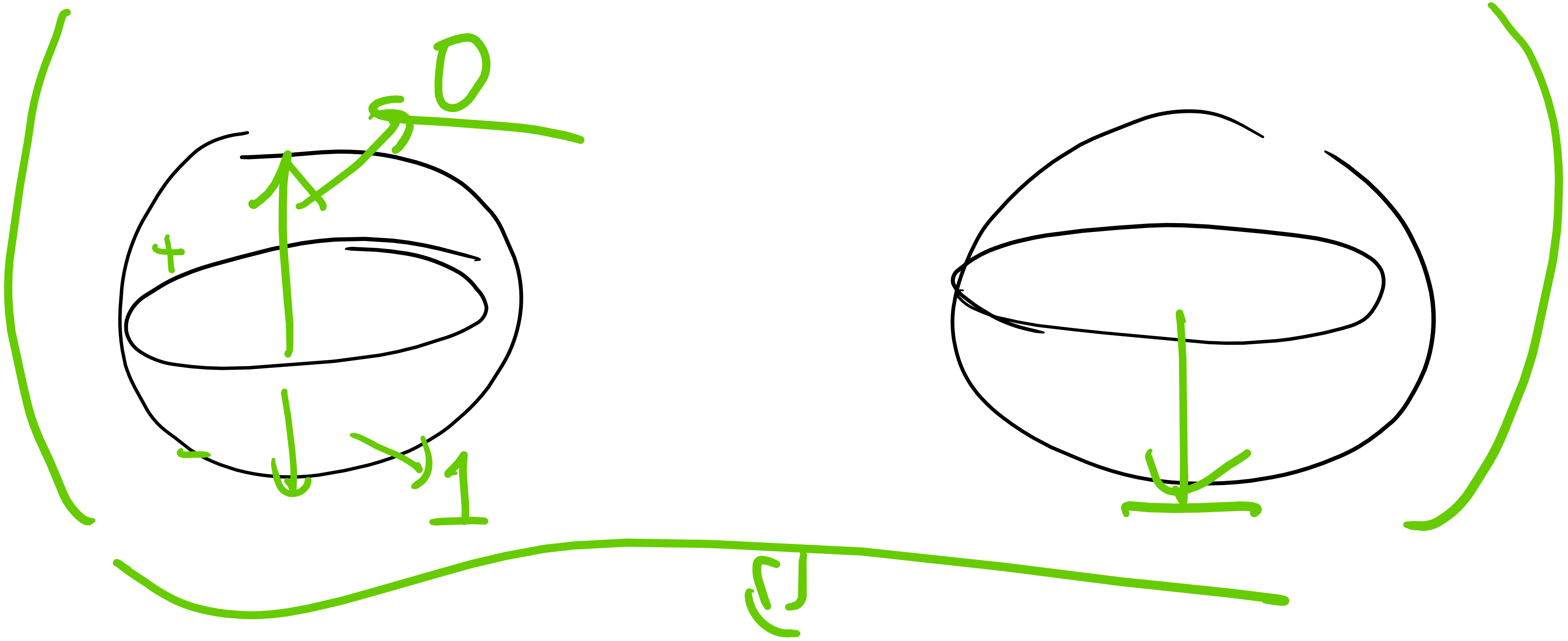
$\Downarrow$

EPR theorem

Einstein

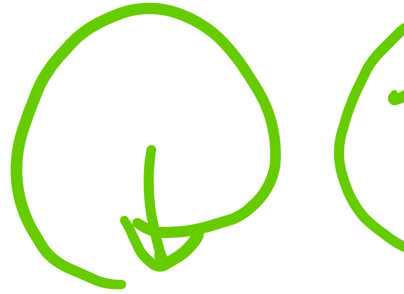
Podolski

Rosen



one basis

$|01\rangle$



$|10\rangle$

Why $\underline{QE}$ is important?

$$|\psi\rangle = \frac{|0\rangle|1\rangle - |1\rangle|0\rangle}{\sqrt{2}}$$

$$\boxed{|0\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}}} \qquad \boxed{|1\rangle = \frac{|+\rangle - |-\rangle}{\sqrt{2}}}$$

$$|0\rangle|-\rangle = \left(\frac{|+\rangle+|-\rangle}{\sqrt{2}}\right) \cdot \left(\frac{|+\rangle-|-\rangle}{\sqrt{2}}\right)$$

$$= \underline{\frac{1}{2}\Big(|++\rangle - |-+\rangle - |+-\rangle - |--\rangle\Big)}$$

$$|1\rangle|0\rangle = \frac{1}{2}\big(|++\rangle - |-+\rangle + |+-\rangle - |--\rangle$$

$$|\psi\rangle = -\frac{1}{\sqrt{2}}\left(|+-\rangle - |-+\rangle\right)$$

Concept of "entangled"

$X \rightsquigarrow$ basis $(|+\rangle, |-\rangle)$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$Z \rightsquigarrow |0\rangle, |1\rangle$

Particle C

Alice

Bob

Particle A		Partle B	
Z	X	Z	X
+	+	−	−
+	−	−	+
−	+	+	−
−	−	+	+

	Allis			Bob		
	a	b	c	a	b	c
N1	+	+	+	−	−	−
N2	+	+	−	−	−	+
N3	+	−	+	−	+	−
N4	+	−	−	−	+	+
N5	−	+	+	+	−	−
N6	−	−	+	+	+	−
N7	−	+	−	+	−	+
N8	−	−	−	+	+	+

* after measurement

only two directions are remained

$$\underbrace{a_i : +}_{\text{Alli's}}, \underbrace{b_i : -}_{\text{Bob's measure}} = N_1 + N_2$$

$$\underbrace{a_i : +}_{\text{Alli's}},\ \underbrace{c_i : -}_{\text{Bob}} \quad \frac{1}{4}(N_1 + N_2)$$

$$N_1 + N_2 = \frac{2}{8} = \frac{1}{4}$$

* give Special situation

if each a & b the Same case

	Allis (a)	Bob (b)
N_3	+	+
N_4	+	+
N_5	−	−
N_6	−	+
N_7	−	−
N_8	−	+

$N_3 + N_4$

$(+ +)$

$(+ +)$

$$N_3 + N_4 \le N_3 + N_4 + N_2 + N_7$$

$$\frac{N_3 + N_4}{N} \le \frac{(N_2 + N_4)}{N} + \frac{(N_3 + N_7)}{N}$$

$$p(+a, +b) \le p(+a, +c) + p(+c, +b)$$

<proof for "Qubit doesn't mean B.I.">

let a qubit

$$\vec{n} = \sin\theta \cdot \cos\phi \cdot \hat{x} + \sin\theta \cdot \sin\phi \; \hat{y} + \cos\theta \; \hat{z}$$

$$\vec{\sigma} \cdot \vec{n} \Rightarrow \begin{cases} |+_n\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\cdot\phi}\sin\frac{\theta}{2}|1 \\ |-_n\rangle = \cos\frac{\theta}{2}|0\rangle - e^{i\cdot\phi}\sin\frac{\theta}{2}|1 \end{cases}$$

"y"

$|\psi\rangle$

$$|\phi\rangle = \frac{|+a\rangle|-a\rangle - |-a\rangle|+a\rangle}{\sqrt{2}}$$

$$P_r(+a;+c\rangle = \boxed{\frac{1}{2}\sin^2\frac{\theta_{ac}}{2}}$$

$$P_r(+a,+b) = \frac{1}{2}\sin^2\left(\frac{\theta_{ab}}{2}\right.$$

$$P_r(c,+b) = \frac{1}{2}\sin^2\left(\frac{\theta_{cb}}{2}\right)$$

$$\underbrace{\sin^2\left(\frac{\theta_{ab}}{2}\right)} \le \underbrace{\sin^2\left(\frac{\theta_{ac}}{2}\right) + \sin^2\left(\frac{\theta_{cb}}{2}\right)}$$

Bellman inequality

$$\boxed{\theta_{ac} = \theta_{cb} = \theta, \quad \theta_{ab} = 2\theta}$$

$$\boxed{\sin^2\theta \cancel{\le} 2\sin^2\left(\frac{\theta}{2}\right)} \qquad 0 < \theta <$$

Revisit "Entanglement"

$|ab\rangle$

$\Downarrow$

bipartite system

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

$|a\rangle \qquad |b\rangle$

$$\boxed{|\alpha_{ij}\rangle} = |a_i\rangle \otimes |b_j\rangle = |a_i\rangle |b_j\rangle = |a_i b_j\rangle$$

$$\langle \alpha_{ij} | \alpha_{kl} \rangle = \langle a_i b_j | a_k b_l \rangle$$

$$= \langle a_i | a_k \rangle \langle b_j | b_l \rangle$$

$$\delta_{ik} \cdot \delta_{jl}.$$

$$|\psi\rangle = \sum_{ij} c_{ij} \cdot |\alpha_{ij}\rangle = \sum_{ij} |a_i b_j\rangle \langle a_i b_j|\psi\rangle$$

$$c_{ij}$$

$$P_r(a_i, b_j) = \underline{\underline{(\langle a_i b_j|\psi\rangle)^2}}$$

a bipartite system

$$|\psi_1\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} = \beta_{00} \quad \beta_{ab}$$

$$|\psi_2\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}} \Rightarrow \beta_{01}$$

$$|\psi_3\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}} \Rightarrow \beta_{10}$$

$$|\psi_4\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}} \Rightarrow \beta_{11}$$

$$\boxed{|\beta_{xy}\rangle = \frac{|0\,y\rangle + \dots}{\sqrt{2}}}$$

$\Downarrow$

entangle...