Quantum Computing - Summary
- for Engineering Purposes-

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Contents

List of Figures

List of Tables

1 Question

Can you answer these?

1. Line

2 Quantum Postulate

Quantum Postulate

1. Postulate 1

$$|\psi\rangle = a \cdot |0\rangle + b\,|1\rangle \tag{1}$$

2. Postulate 2 (A be a Hermite opeator)

$$A = \sum_i \lambda_i \cdot |u_i\rangle \langle u_i| \tag{2}$$

3. Postulate 3 - measurement

$$P = \langle\psi|P|\psi\rangle = Tr(P \cdot |\psi\rangle \langle\psi|) \tag{3}$$

4. Postulate 4 - dynamics (Schrodinger equation)

$$ih \cdot \frac{\partial}{\partial t} \cdot |\psi_t\rangle = H \cdot |\psi_o\rangle \tag{4}$$

$$|\psi_t\rangle = e^{-\frac{iHt}{h}} \cdot |\psi_0\rangle \tag{5}$$

3 State & System

3.1 State & System

$|\psi\rangle$

$$0! \rightarrow |\psi\rangle \rightarrow |\psi_t\rangle \tag{6}$$

with Hadamard transpormation (Hadamard gate)

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{7}$$

Let a $|\psi\rangle$ be

$$|\psi\rangle = \alpha \cdot |0\rangle + \beta \cdot |1\rangle \tag{8}$$

Then,

$$H \cdot |\psi\rangle = \alpha \cdot H \cdot |0\rangle + \beta \cdot H \cdot |1\rangle \tag{9}$$

3.2 Born's rule

Provided by Max Born

Let a measure P_i be a project measure with respect to an operator A with $|\psi\rangle$

$$P_i = \langle\psi|A|\psi\rangle \tag{10}$$

If A has a spectrum of λ_i, then

$$P_i = \langle\psi|\lambda_i\rangle \langle\lambda_i|\psi\rangle \tag{11}$$

Relationship with "Trace", in a pure state

$$P_i = \langle\psi|A|\psi\rangle = tr(|\psi\rangle \langle\psi| A) \tag{12}$$

3.3 pure state Vs mixed state

Let ρ be

$$\rho = \sum_{i=0}^{n} q_i \cdot |\psi_i \times \psi_i| \tag{13}$$

in pure state,

$$tr(\rho^2) = 1 \tag{14}$$

3.4 inner product

a measurement of $\langle\psi|$, in terms of $\langle\phi|$

$$\langle\phi|\psi\rangle \tag{15}$$

3.5 outer product

1. interpretation
   1) a state by a state
   2) an operator

$$|\psi\rangle \langle\psi| \tag{16}$$

is a matrix (a state by a state :operator)

Let A be a operator

$$A = A \cdot I = A \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = A \cdot \begin{pmatrix} \langle u_i|u_i\rangle & \langle u_i|u_j\rangle \\ \langle u_j|u_i\rangle & \langle u_j|u_j\rangle \end{pmatrix} \tag{17}$$

$$\begin{pmatrix} \langle u_i| A_{ii} |u_i\rangle & \langle u_i| A_{ij} |u_j\rangle \\ \langle u_j| A_{ji} |u_i\rangle & \langle u_j| A_{jj} |u_j\rangle \end{pmatrix} \tag{18}$$

4 Operator & Dynamics

4.1 Spectrum Analysis of an Operator

$$A = \sum_{i=0}^{m} a_i \cdot |u_i\rangle \langle u_i| \tag{19}$$

4.2 Pauli transformation : a special Unitary Operator

$$I, X, Y, Z \tag{20}$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{21}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{22}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \tag{23}$$

Phase matrix

$$P(\alpha) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i \cdot \alpha} \end{pmatrix} \tag{24}$$

in general,

$$P = \begin{pmatrix} 1 & 0 \\ 0 & z \end{pmatrix} \tag{25}$$

4.3 Hermitian Operator

H

$$H = H^{+} \tag{26}$$

4.4 Unitary Operator

U

$$U \cdot U^{+} = I \tag{27}$$

4.5 Basis change

Change from $|psi\rangle$ to $|phi\rangle$

Let

$$|\psi\rangle = \sum_i a \cdot |u_i\rangle \langle u_i| \tag{28}$$

and

$$|\phi\rangle = \sum_i a' \cdot |v_i\rangle \langle v_i| \tag{29}$$

Then, the Unitary operator (U) is

$$U = \begin{pmatrix} \langle v_1|u_1\rangle & \langle v_1|u_2\rangle \\ \langle v_2|u_1\rangle & \langle v_2|u_2\rangle \end{pmatrix} \tag{30}$$

So,

$$|\phi\rangle = U \cdot |\psi\rangle \tag{31}$$

$$|\psi\rangle = |\phi\rangle \cdot U^{+} \tag{32}$$

5 Measurement

5.1 Expectation

$$< A >= \langle\psi|A|\psi\rangle = Tr(A\,|\psi\rangle\langle\psi|) = Tr(A \cdot \rho) \tag{33}$$

6 To the Complex system : Tensor product

6.1 Tensor product

$$|00\rangle = |0\rangle \otimes |0\rangle \tag{34}$$

6.2 Density

$$\rho = |\psi\rangle \langle\psi| \tag{35}$$

Density operator

1. $$\rho = \rho^{+}(H) \tag{36}$$

2. $$Tr(\rho) = 1 \tag{37}$$

3. It is a kind of Probability!
$$\langle u|\rho|u\rangle \geq 0 \tag{38}$$

4. (By Born's rule)
$$\rho_t = U \cdot \rho_0 \cdot U^{+} \tag{39}$$

6.3 Project measurement (Von Neumann measurement)

P

Density operator

1. $$P = P^{+}(H) \tag{40}$$

2. $$P^2 = 1 \tag{41}$$

3. Orthogonal
$$P_1 \cdot P_2 |\psi\rangle = 0 \tag{42}$$

4. a Probability
$$\sum_i P_i = I \tag{43}$$

$$P_i = \sum_j |\langle u_j|\psi\rangle|^2 = \langle\psi|P_i|\psi\rangle = Tr(P_i \cdot |\psi\rangle\langle\psi|) = Tr(P_i \cdot \rho) \tag{44}$$

6.4 Collpase of the wave function

"The wave function" $\rightarrow$ a state $|\psi\rangle$

"Collpase" $\rightarrow$ "a change of basis"

$$|\psi_0\rangle \rightarrow (dynamics)\,|\psi_t\rangle \rightarrow (measurement)\,\left|\psi'\right\rangle \tag{45}$$

Then,

$$\left|\psi'\right\rangle = \frac{P \cdot |\psi_t\rangle}{\sqrt{\langle\psi|P|\psi\rangle}} \tag{46}$$