<April 04, Monday>

* State & operator

$|\psi\rangle$ $\qquad$ A

$A = A^{\dagger} \Rightarrow$ Hermite operator

$A \cdot A^{\dagger} = I \Rightarrow$ Unitary operator

$$|\psi_0\rangle \xrightarrow{U} |\psi_t\rangle$$

<Quantum gate>

* "general gate"

X_1	X_2	Y (OR gate)
1	1	1
1	0	1
0	1	1
0	0	0

$$|\psi\rangle \xrightarrow{A} |\psi_t\rangle$$

$$U \rightsquigarrow U^{+}$$

general computation $\Rightarrow$ irreversible

QC $\Rightarrow$ reversible

Ket vector $|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix} = a|0\rangle + b|1\rangle$

bra vector $\langle\psi| = (a \quad b)$

$\langle\psi|\phi\rangle \Rightarrow$ Scalar

$|\phi\rangle\langle\phi| \Rightarrow$ operator

* $A \rightsquigarrow$ Spectrum Analysis

$$\boxed{A \cdot \boxed{\nu} = \lambda \cdot \boxed{\nu}}$$

Vector → Quantum State

Vector → Quantum State

$$\boxed{A = \sum_{i=1}^{n} \lambda_i \, |u_i\rangle\langle u_i|}$$

$Tr \to$ Trace $\Rightarrow$ " Probability "

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix},$$

$$Tr(A) = a + e + i$$

$$A = \sum_{i=1}^{3} \lambda_i |u_i\rangle\langle u_i|$$

$$Tr(A) = \sum_{i=1}^{3} \langle u_i | A$$

$$\sum \langle u_i | \sum \lambda_i |u_i\rangle\langle u_i|$$

$$= \sum \langle u_i | A | u_i \rangle$$

ex) $A = 2i\,|0\rangle\langle 0| + 3|0\rangle\langle 1| - 2|1\rangle\langle 0| + 4|1\rangle\langle 1|$

$$\Downarrow$$

$$\begin{pmatrix} 2i & 3 \\ -2 & 4 \end{pmatrix}$$

$$Tr(A) = \sum_{i=0}^{1} \langle u_i | A | u_i \rangle$$

$$= \langle \overset{0}{u_0} | A | u_0 \rangle + \langle \overset{1}{u_1} | A | u_1 \rangle$$

$$= \langle 0 | A | 0 \rangle + \langle 1 | A | 1 \rangle$$

$$= 2i + 4$$

* operator

two state

$|\psi\rangle$

$|\phi\rangle$

$|\phi\rangle\langle\psi|$

$Tr(|\phi\rangle\langle\psi|)$

$= \langle\psi|\phi\rangle$

X : $X|0\rangle \Rightarrow |1\rangle$

$X \cdot X^T = I$

$X = X^T$

Y

$Y \cdot Y^T = I$

$Y = Y^T$

Z :

matrix

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$Tr(Z) = 0$

⋮

① $\mathrm{Tr}(A|\psi\rangle\langle\phi|) = \langle\phi|A|\psi\rangle$

② $\mathrm{Tr}(A) = \sum_{i=1}^{n} \lambda_i$

eigen value

③ $\mathrm{Tr}(|\phi\rangle\langle\psi|) = \langle\psi|\phi\rangle$

Trace characteristics

< Proof of $\mathrm{Tr}(A|\phi\rangle\langle\psi|) = \langle\psi|A|\phi\rangle$ >

for your midterm

$$\sum_{i=1}^{n} \langle u_i | \boxed{} | u_i \rangle = \sum_{i=1}^{n} \langle u_i | A | \phi \rangle \langle \psi | u_i \rangle$$

$$= \sum_{i=1}^{n} \langle \psi | u_i \rangle \langle u_i | A | \phi \rangle$$

$$\langle \psi | A | \phi \rangle = \langle \psi | \underbrace{\sum_{i=1}^{n} |u_i\rangle\langle u_i|}_{I} A | \phi \rangle$$

* meaning

We have a Q.S. $|\psi\rangle$.

$$\langle A \rangle = \langle \psi | A | \psi \rangle$$

$\Downarrow$

Expectation

$$|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle \quad \left(\frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}\right) \quad \begin{pmatrix} \sqrt{\frac{2}{3}} \\ \sqrt{\frac{1}{3}} \end{pmatrix}$$

$$\Downarrow$$

EX $\Rightarrow \langle \psi | X | \psi \rangle = \langle X \rangle$ $\quad \frac{2\sqrt{2}}{3}$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \left(\frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle \right| X \left| \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle \right)$$

$$\begin{cases} X|0\rangle = |1\rangle \\ X|1\rangle = |0\rangle \end{cases} \qquad \left(\frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle \right| \sqrt{\frac{2}{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle$$

<Taylor Series>

A

$$f(A) = \sum_{n=0}^{\infty} a_n \cdot \boxed{A^n}$$

operator

$$A = \sum_i a_i |u_i\rangle\langle u_i|$$

Then

$$f(A) = \sum_{i=1}^{\infty} f(a_i) \boxed{|u_i\rangle\langle u_i|}$$

$$\begin{cases} H \rightsquigarrow \text{Hermite operator} \\ U \rightsquigarrow \text{Unitary operator} \end{cases}$$

$$\boxed{H = \sum_i \phi_i \left(|u_i\rangle\langle u_i| \right)}$$

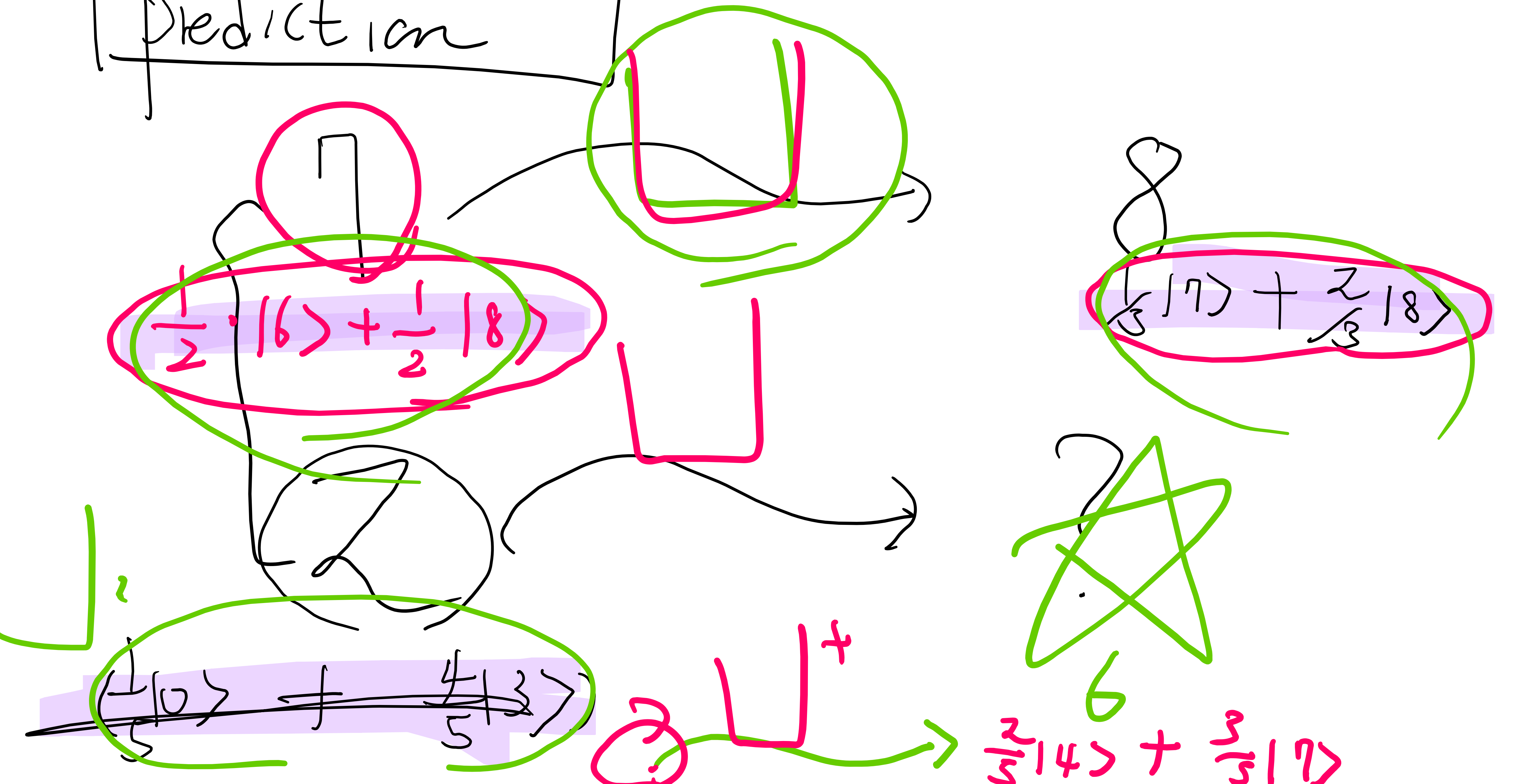
$$\boxed{U = e^{i \cdot \varepsilon \cdot H}}$$

$$= e^{i \varepsilon \phi_n} \cdot |u_n\rangle\langle u_n|$$

$$A = \star \, |u_i\rangle\langle u_i|$$

$$f(A) = f(\star) \cdot \underline{|u_i\rangle\langle u_i|}$$

Prediction


7
$\frac{1}{2}|6\rangle + \frac{1}{2}|8\rangle$
2
$\frac{1}{5}|0\rangle + \frac{4}{5}|3\rangle$
8
$\frac{1}{3}|7\rangle + \frac{2}{3}|8\rangle$
2
6
U^{+}
$\frac{2}{5}|4\rangle + \frac{3}{5}|7\rangle$

$$|\psi\rangle = \sum_i \star \, |u_i\rangle\langle u_i|$$

basis change.

$$|\psi'\rangle = \sum_i \triangle \, |v_i\rangle\langle v_i|$$

$$U = \begin{pmatrix} \langle v_1|u_1\rangle & \langle v_1|u_2\rangle \\ \langle v_2|u_1\rangle & \langle v_2|u_2\rangle \end{pmatrix}$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$$\psi = |0\rangle + |1\rangle \qquad U = \begin{pmatrix} \langle +|0\rangle & \langle +|1\rangle \\ \langle -|0\rangle & \langle -|1\rangle \end{pmatrix} \qquad \frac{1}{\sqrt{2}}$$

$$\psi' = |+\rangle + |-\rangle$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle \longrightarrow \ ?$$

$$U\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix}$$

$$= \frac{1}{\sqrt{6}}\begin{pmatrix} 1 + \sqrt{2} \\ 1 - \sqrt{2} \end{pmatrix}$$

?

U

$\frac{1}{2}|1-\sqrt{3}\rangle + \frac{1}{2}|1$

$\neq \phi$

$\boxed{\phi \cdot U}$

$U^{-1} = U^{\dagger} = U$

Quantum deep learning.

$$\underbrace{A}_{\text{operator}} \longrightarrow \underbrace{|u_n\rangle}_{\text{eigen value}}$$

$\Downarrow$

Spectrum Analysis

* $\boxed{\text{Probability}}$

$$\langle \psi | A | \psi \rangle$$

$$A \sim |0\rangle\langle 0|$$

$$= |0\rangle$$

$$\boxed{A = |0\rangle\langle 0|}$$

$$|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$$

$$P_0 = \langle \psi | \, |0\rangle\langle 0| \, |\psi\rangle = \frac{1}{3}$$

$$|\psi_0\rangle \xrightarrow{\square} |\psi_t\rangle$$

$$\begin{cases} P_{\phi_1} = \alpha^2 \\ P_{\phi_2} = \beta^2 \end{cases}$$

$$f\left(\alpha|\phi_1\rangle + \beta|\phi_2\rangle\right)$$

$$|\phi_0\rangle \longrightarrow ?$$

$$f(\alpha)|\ \rangle + f(\beta)\ \rangle$$