Takeaways from Quantum Computing

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1 Main Reference

> **Main Reference**
>
> - Aram W. Harrow, Avinatan Hassidim, Seth Lloyd, Quantum algorithm for linear systems of equations, Physical Review Letters, Vol 103, No. 150502, 2009, https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.103.150502
>
> - Hector Jose Morrel Jr, Anika Zaman, Hiu Yung Wong, Step-by-step HHL algorithm walkthrough to enhance the understanding of critical quantum computing concepts, 2021, https://arxiv.org/abs/2108.09004

2 Takeaways

> **Question & Takeaways**
>
> 1. Role of H
>
> 2. Quantum Phase Transformation & QFT
>
> 3. Ancilla register
>
> 4. Normalization for Amplitudes

3 HHL algorithm

3.1 Misunderstanding & Concepts

HHL algorithm can't find the solution of linear systems. How does it can find the solutions?

Three gates : B registers, Clock registers, Anchilla registers

3.2 Basic Procedures

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Procedures

1. $|\psi_1\rangle$: 1) Preparation of $|b\rangle$

$$|b\rangle |0\cdots 0\rangle |0\rangle \tag{1}$$

2. $|\psi_2\rangle$: 1) Superposition of A & 2) A $\to e^{2\pi i\theta}$

$$|b\rangle H^n \otimes |0\cdots 0\rangle |0\rangle \tag{2}$$

$$B_1 |u_1\rangle H^n \otimes |0\cdots 0\rangle |0\rangle + B_2 |u_2\rangle H^n \otimes |0\cdots 0\rangle |0\rangle + \cdots \tag{3}$$

3. $|\psi_3\rangle$: 1) Phase estimation of $|u_i\rangle$ for b

$$U|u_i\rangle = e^{2\pi i\theta} |u_i\rangle \tag{4}$$

why? It is from Schrodinger equation

$$B_i |u_i\rangle \frac{1}{2^{n/2}} \sum_{k=0}^{2^n-1} e^{\pi i\phi k} |k\rangle |0\rangle \tag{5}$$

4. $|\psi_4\rangle$: 1) QFT of $|\psi_3\rangle$
   Unitary Check

5. $|\psi_5\rangle$: 1) Ancilla bit rotation
   Rotation w.r.t. Y axis

$$|0\rangle \to \sqrt{1 - \frac{C^2}{\bar{\lambda}_j^2}} |0\rangle + \frac{C^2}{\bar{\lambda}_j^2} |1\rangle \tag{6}$$

$$B_i |u_i\rangle \frac{1}{2^{n/2}} \sum_{k=0}^{2^n-1} e^{\pi i\phi k} |k\rangle \sqrt{1 - \frac{C^2}{\bar{\lambda}_j^2}} |0\rangle + \frac{C^2}{\bar{\lambda}_j^2} |1\rangle \tag{7}$$

6. $|\psi_6\rangle$: 1) Measure/ Take for only parts of $|1\rangle$ in ancilla bit

7. $|\psi_7\rangle$: 1) Invere QFT, Inverse $|\psi_4\rangle$

8. $|\psi_8\rangle$: 1) Inverse $|\psi_3\rangle$

9. $|\psi_9\rangle$: 1) Inverse H for $|\psi_2\rangle$

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4 Practice

Try it with your HHL algorithm.

Practice

$$x_1 + 2x_2 + 3x_3 = 9 \tag{8}$$

$$4x_1 + 5x_2 + 6x_3 = 24 \tag{9}$$

$$3x_1 + x_2 - 2x_3 = 4 \tag{10}$$