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Single Workstation Factory Model

$$WIP_q = \lambda \cdot \left(\frac{U}{1-U} \right) \cdot E[S] \cdot \left(\frac{1+C^2[S]}{2} \right)$$

$$CT_q = \left(\frac{U}{1-U} \right) \cdot E[S] \cdot \left(\frac{1+C^2[S]}{2} \right)$$

HYUNSOO LEE

Review (1)

- Analyzing Procedures for Simple system
  - Transition diagram
  - Balance equation
  - Calculation of probability
  - Calculation of WIP
  - Estimation of TH
  - Calculation of Cycle time
  - Analysis of other performance indexes

Review (2)

- Utilization factor
  - E[busy in server]

Homework

- Due :
  - CT in system CT_S

U	M/M/1	M/M/2	M/M/3
0.5			
0.6			
0.7			
0.8			
0.9			

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Contents

- Expansion to general cases
  - Erlang Distribution
  - $M/E_K/N$
  - Coxian Distribution
  - M/G/N

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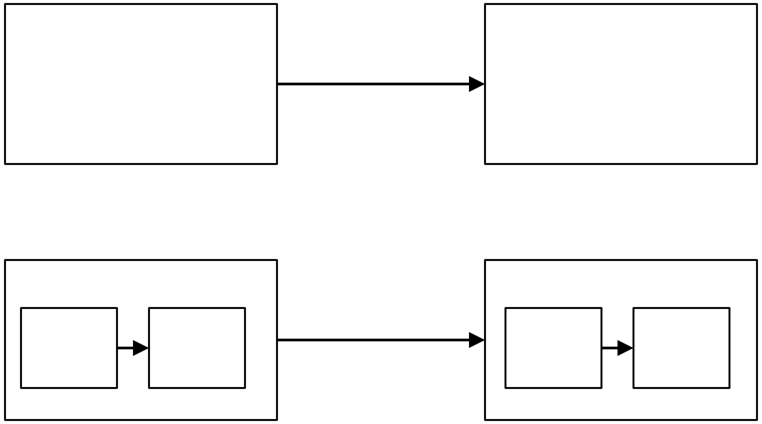
Erlang Distribution (1)

- Cases



- Question
  - Total Cycle time → Sum of each cycle time → ?
  - Which system is better between M/M/1 and $M/E_K/1$?

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Erlang Distribution (2)

- Analysis of Erlang distribution
  - Exponential distribution
  - Coefficient of Variance

$$C^2[T] = \frac{V[T]}{E[T]^2}$$

  - Erlang (K)

Erlang Distribution (3)

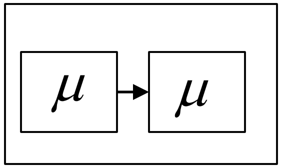
- Calculation of CoV
  - M/M/1
  - M/E_K/1

$$E[2X] = 2E[X]$$

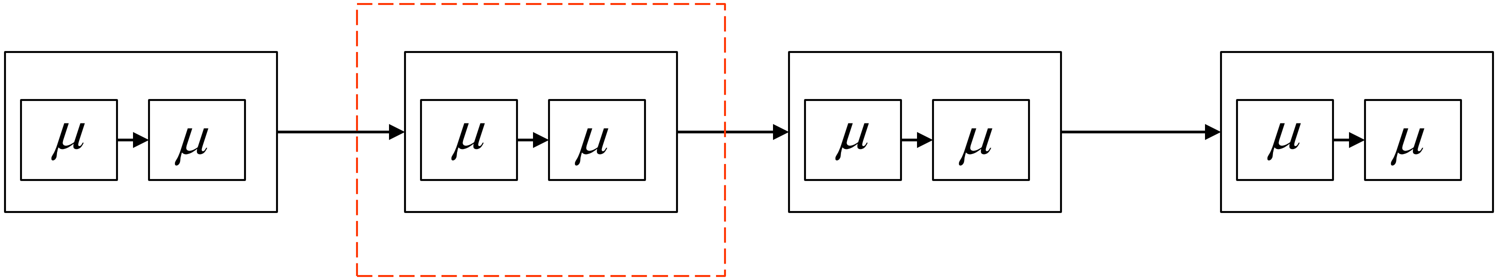
$$V[2X] = 4V[X]$$

M/E$_2$/1

- Case



- Transition Diagram



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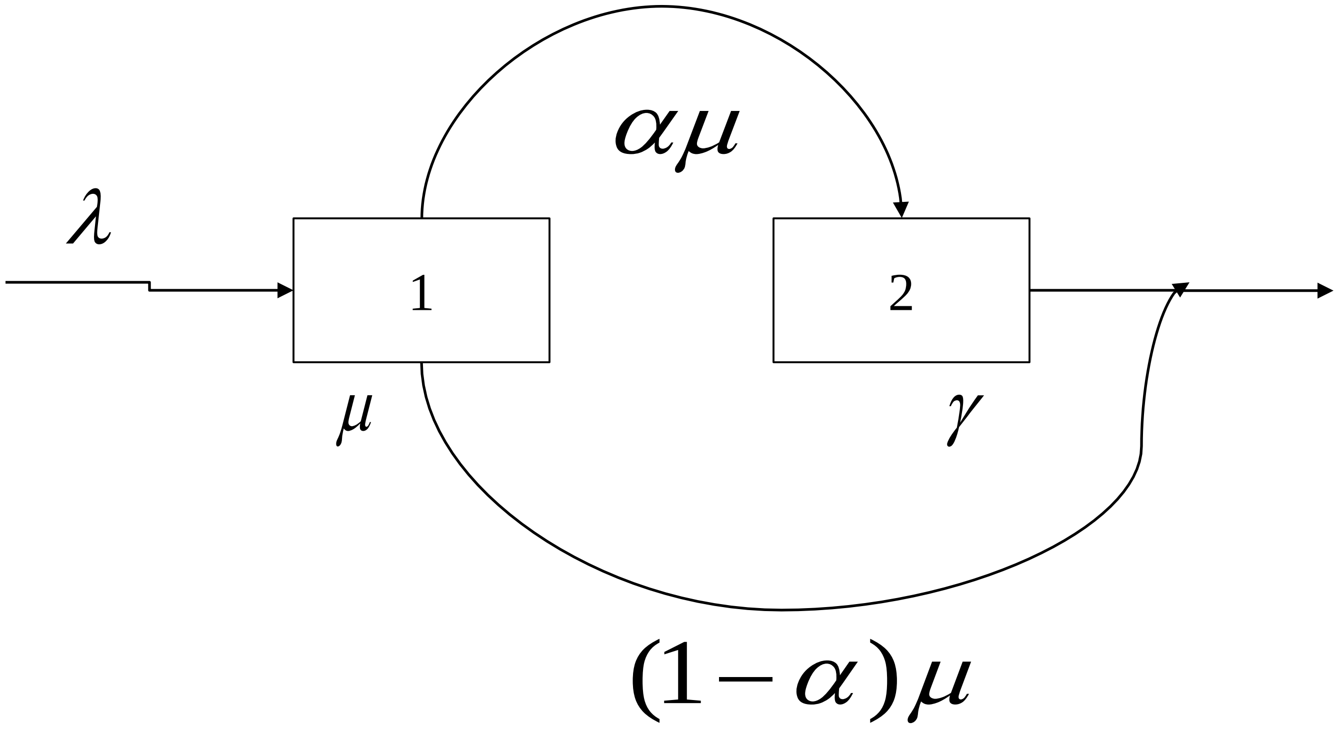
Coxian Distribution (1)

- Cases $M / Cox(\mu, \alpha, \gamma) / 1 / 3$



Coxian Distribution (2)

- Exercise

$$\lambda = 4 / hr$$

$$\mu = 6 / hr$$

$$\gamma = 5 / hr$$

$$\alpha = 1/10$$

  - WIP

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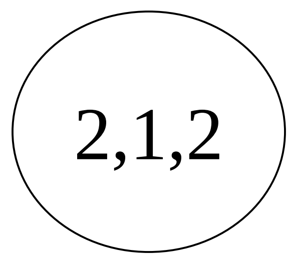
Coxian Distribution (3)

- M / Cox (μ,γ,α) / 2
  - Transition Diagram



- Hint
  - WIP
  - Arrival
  - Service

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Coxian Distribution (3)

- Control model
  - E[S] $$E[S] = \frac{1}{\mu} + \alpha\frac{1}{\gamma}$$
  - Control , Given E[S] and CoV $C^2[S]$
    - Using “Moment methods”
      - Case 1: $C^2[S] > 1$

$$\mu = \frac{2}{E[S]} \qquad \gamma = \frac{1}{E[S]\cdot C^2[S]} \qquad \alpha = \frac{1}{2\cdot C^2[S]}$$

      - Case 2: $\frac{1}{2} \le C^2[S] \le 1$

$$\mu = \frac{1}{E[S]\cdot C^2[S]} \qquad \gamma = \frac{2}{E[S]} \qquad \alpha = 2(1 - C^2[S])$$

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M/G/1 (1)

- Pollaczek and Khintchine (1931)

$$E[N_q] = \frac{\left(\frac{\lambda}{\mu}\right)^2 + \lambda^2 \sigma_s^{2}{}^{2}}{2\left(1 - \frac{\lambda}{\mu}\right)}$$

  - WIP →

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M/G/1 (2)

- Expansion to general Eq.

$$E[N_q] = \frac{\left(\frac{\lambda}{\mu}\right)^2 + \lambda^2 \sigma_s^2}{2\left(1 - \frac{\lambda}{\mu}\right)}$$

$$E[N_q] = \frac{\lambda U E[S] + \lambda^2 \sigma_s^2}{2(1-U)}$$

$$= \frac{\lambda U E[S] + \lambda^2 E[S]^2 C^2[S]}{2(1-U)}$$

$$= \frac{\lambda U E[S] + U \lambda E[S] C^2[S]}{2(1-U)}$$

$$= \frac{U \lambda E[S]}{(1-U)} \cdot \left(\frac{1 + C^2[S]}{2}\right)$$

$$CT_q = \left(\frac{U}{1-U}\right) \cdot E[S] \cdot \left(\frac{1 + C^2[S]}{2}\right)$$

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M/G/1 (3)

- Comparison with M/M/1

$$CT_q = \left(\frac{U}{1-U} \right) \cdot E[S] \cdot \left(\frac{1+C^2[S]}{2} \right)$$

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M/G/C

- Generalization to M/G/C

$$CT_q = \left(\frac{U}{1-U}\right) \cdot E[S] \cdot \left(\frac{1 + C^2[S]}{2}\right) \cdot \left(\frac{U^{\sqrt{2C+2}-2}}{C}\right)$$

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G/G/C

- Generalization to G/G/C

$$CT_q = \left(\frac{U}{1-U}\right) \cdot E[S] \cdot \left(\frac{C^2[A]+C^2[S]}{2}\right) \cdot \left(\frac{U^{\sqrt{2C+2}-2}}{C}\right)$$