<Oct 10th>

1. 10.19th → No class

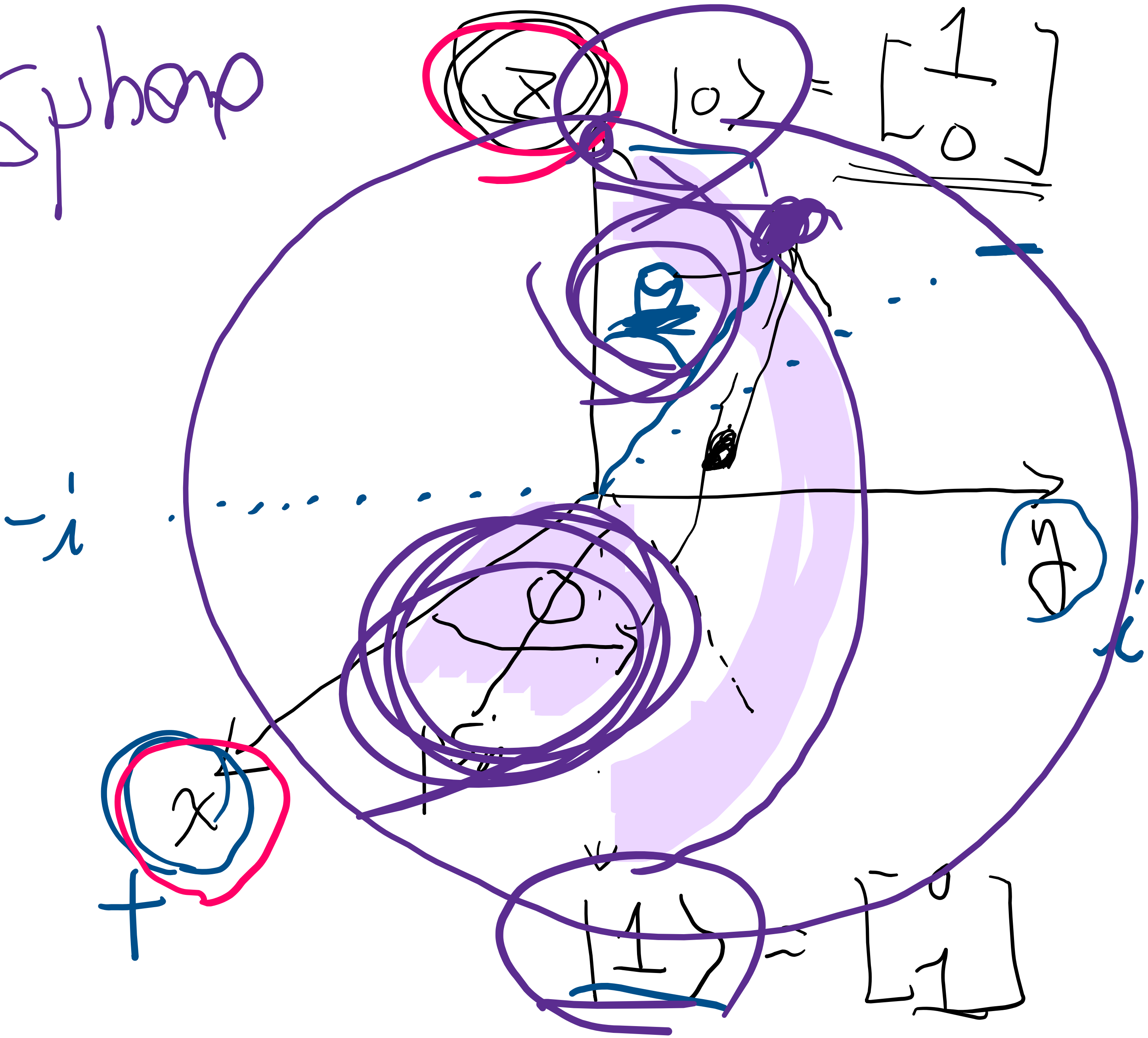
10.24th ⇒ offline class

Quantum
Mechanics

||

Quantum
Computing → qubit ⇝ bloch
sphere.

Sphere



$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \ldots$

phi

$e^{i\phi}$

Special Brownian motion.

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

$\Downarrow$

Qubit

$0 \le \theta \le \pi$

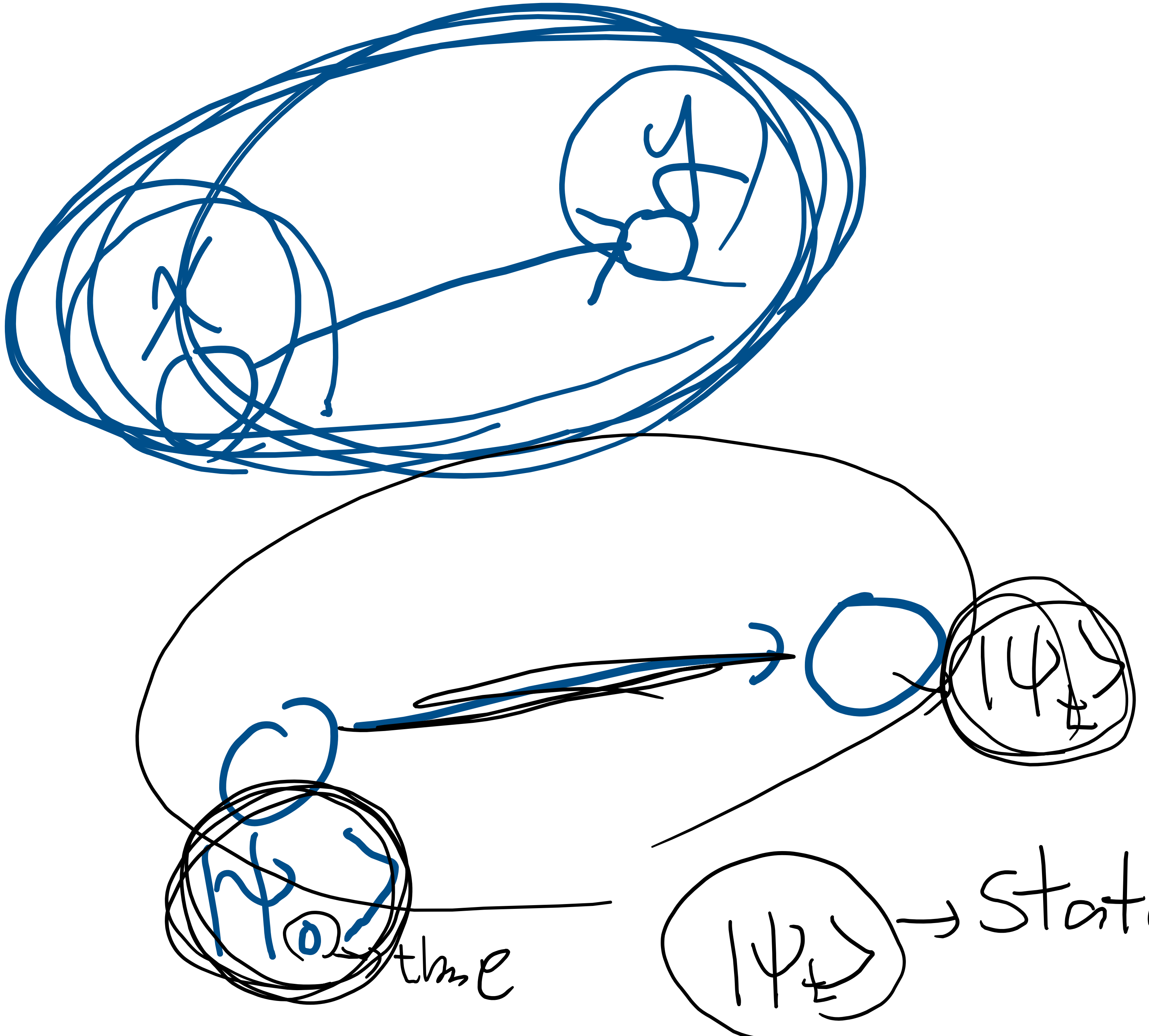
$0 \le \phi \le 2\pi$

Q, ϕ

two/multiple classical bits

① rotation #1 w.r.t $x(t)$

② rotation #2 w.r.t



$|\psi_t\rangle \to$ state at the t

$$|\psi_0\rangle \longrightarrow |\psi_t\rangle$$

Quantum dynamics

$$|\psi_t\rangle = H\,|\psi_0\rangle$$

future current

Prediction
forcasting

$$|\psi_t\rangle = H \cdot |\psi_0\rangle$$

$t \Rightarrow$ change at small time.

$$|\psi_t\rangle = H(t) \cdot \underbrace{|\psi_0\rangle}_{\text{initial status}}$$

$$\frac{\partial x}{\partial t} = 4x^2 + \sin x + 4$$

$$\frac{\partial x}{\partial t} = 5x^2 + \cos t + 6 + B_t$$

stochastic differential equation

Brownian motion

SDE

$\Downarrow$

Quantum mechanics -based equation

$\Downarrow$

dynamic network

*

Quantum Mechanics

~~Quantum Computing~~

$\cos\frac{\theta}{2}|\psi_0\rangle + i\sin\frac{\theta}{2}|\psi_1\rangle$ Qubit

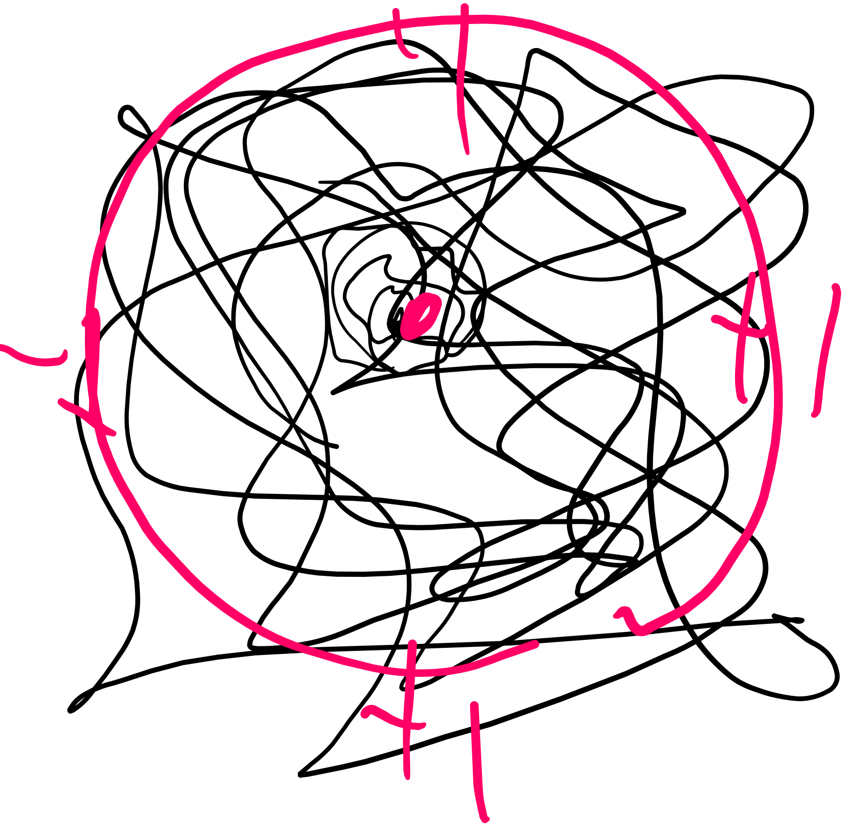
$\frac{1}{2}$

$* \; B_t$

"Brownian motion at time t"

$$\begin{cases} B_0 \sim 0 \\ B_t \sim N(0, t) \\ B_{t+s} - B_t \sim N(0, s) \end{cases}$$

$t=0$ $t=1$ $t=10$



$N(0,t)$

$X \sim$ Gaussian dist

$Y \sim$ Gaussian dist

$X \sim Y \Rightarrow U$

$T \to$ time

$$\int_0^T B_s\, dB_s$$ Brownian motion

$$= \frac{1}{2} B_T^2 - \frac{1}{2} T$$

☆ Ito integral

B_t

B_{t+s}

B_{t+st} 0

$$\frac{\partial Q}{\partial t} \approx \int \Delta + \int 0 + \int \cancel{A} B_t + \int 0 B_{t+s}$$

$$\boxed{f(t, B_t)}$$

time noise

drone's position $\Rightarrow f$

1. time (t)
2. K (driving pattern)
3. β_t

$f(t, K, \beta_t, \dots)$

$\frac{\partial f}{\partial t}$

$$\left\{\begin{array}{l} \text{time} = \underline{100} \Rightarrow (x, y, z) \\ \text{time} = 90 \Rightarrow (x, y, z) \end{array}\right.$$

$$\cancel{\frac{\partial f}{\partial t} = \left\{ k + \frac{\partial x}{\partial t} + \frac{\partial y}{\partial t} + \frac{\partial z}{\partial t} + t^3 \cdot x \cdot f \cdot B \right.}$$

model

$f_{1,1}$

$$\boxed{E\left[\int_0^T B_s\, dB_s\right]}$$

$\Rightarrow$ expectation of Ito integral

$$= E\left[\frac{1}{2}B_t^2 - \frac{1}{2}T\right] \qquad \frac{1}{2}B_t^2 - \frac{1}{2}T$$

constant

$$\Rightarrow E[B_t^2] - \frac{1}{2}T$$

$$E\left[\int_0^T B_s B_s\right]$$

$$= \boxed{\frac{1}{2}E[B_T^2] - \frac{1}{2}T}$$

$$\boxed{\text{Variance of } B_T}$$

$$= \frac{1}{2}T - \frac{1}{2}T = 0$$

$$\boxed{\int_0^T B_s^n \cdot dB_s}$$

$$\boxed{f(B_t) = \frac{1}{n+1} \cdot B_t^{n+1}}$$

$$df(B_t) =$$

Ito-lemma

$$f(B_t)$$

$$= df(B_t)$$

$$= f'(B_t)\,dB_t + \frac{1}{2} f''(B_t) \cdot dB_t^2$$

$$= f'(B_t) \cdot dB_t + \frac{1}{2} f''(B_t) \cdot dt$$

$$B_t^2 \Rightarrow t$$

$f(x, t, B_t)$

$df = \frac{\partial f}{\partial x} \cdot dx + \frac{\partial f}{\partial t} dt + \frac{\partial}{\partial}$

$f(x, t, B_t) \Rightarrow f(x, t, B_t)$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial B_t} \cdot dB_t$$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial B_t} dB_t + \frac{1}{2} \frac{\partial^2 f}{\partial B_t^2} dt$$

$$= \frac{\partial f}{\partial x} dx + \left(\frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial B_t^2} \right) dt + \frac{\partial f}{\partial B_t} dB_t$$

$$f(B_t) = \frac{1}{n+1} B_t^{n+1}$$

$$df = \frac{\partial f}{\partial B_t} dB_t + \frac{1}{2}\frac{\partial^2 f}{\partial B_t^2} \cdot dB_t^2$$

$$dt$$

$$= \frac{1}{2}\frac{\partial^2 f}{\partial B_t^2} \cdot dt + \frac{\partial f}{\partial B_t} \cdot dB_t$$

$$n \cdot B_t^{n-1} \qquad B_t^{n}$$

$$df = \frac{1}{2} \cdot n \cdot B^{n-1} \cdot dt$$

$$+ B_t^{n} d$$

$$\boxed{\int df = \int \frac{1}{2} n \cdot B^{n-1} dt + \int B^n \cdot dB_t}$$

add with every time

$$\left[\boxed{\frac{1}{n+1} B_t^{n+1}}\right] = \left[\frac{1}{2}\right] n \left[\int B_t^{n-1} \cdot dt\right] + \left[\boxed{\int^T B_t^n dB_t}\right]$$

$$E\left[\frac{1}{n+1} \cdot B_t^{n+1}\right] = \frac{1}{2} n \cdot E\left[\int B_t^{n-1} dt\right]$$

0

$$\frac{1}{n+1} E\left[B_t^{n+1}\right] = \frac{n}{2} E\left[\int_0^T B_t^{n-1}\, dt\right]$$

$$6\int_0^T t\,dt = 6\cdot\left[\frac{1}{2}t^2\right]_0^T = 3T^2$$

$$E\left[B_t^{n+1}\right] = \frac{n(n+1)}{2} E\left[\int_0^T B_t^{n-1}\, dt\right] = 6\int_0^T t\,d$$

$3T^2$

$$E\left[B_t^4\right] = \frac{3\cdot 4}{2} E\left[\int_0^T B_t^2\, dt\right] = \frac{3\cdot 4}{2}\int_0^T E$$

① Prediction

② Variables B_t

→ model (f) deep learn

③ differential equation. (B_t)

④ ito integral & $E(\ )$

real model with B_t & calculate.

⑥ Random

$$\cancel{f(t, B_t) = t + e^{B_t}}$$

$$e^{B_t}$$

$$df = \left(\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial B_t^2}\right) dt + \frac{\partial f}{\partial B_t} dB_t$$

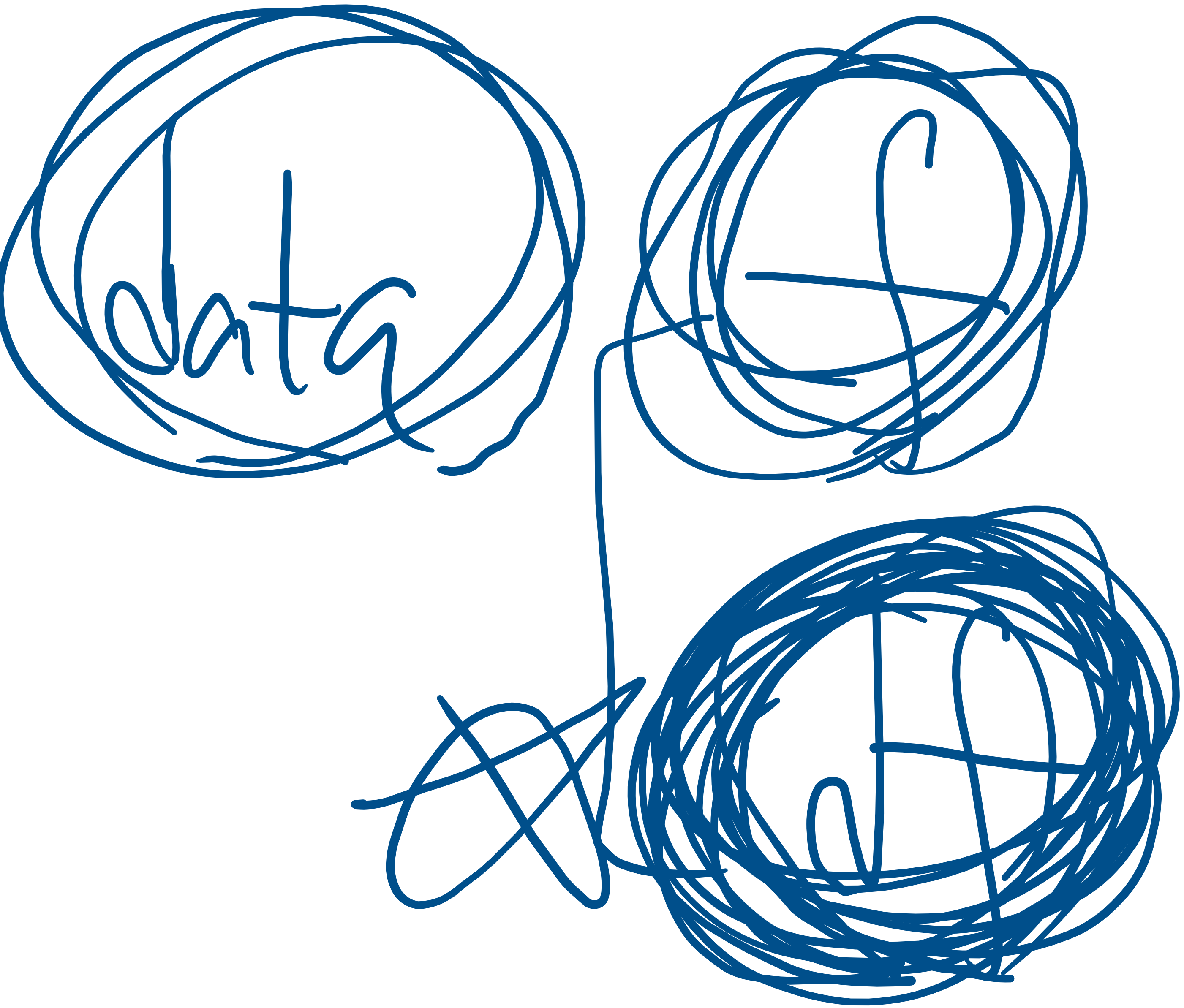
f

$t \qquad B_t$

$1 \qquad e^{B_t} \qquad e^{B_t}$

$$df = (1 + e^{B_t})\,dt + e^{B_t}\,dB_t$$

$$df = \left(\ \right) dt + \frac{\partial f}{\partial B_t} \cdot dB_t + \frac{1}{2}\frac{\partial^2 f}{\partial B_t^2}\, d$$


data

$$\cancel{f(x, y, z, t, B_t)}$$

$$df(x, y, z, t, B_t) = 0\,dx$$

9 5 4, 2 7

10 8

$\int 4x + 3dy + 6dx$

$0 + 0 + 3x$

$$f(t, B_t) = t \cdot B_t$$

$$\Downarrow$$

$$df = \underbrace{\frac{\partial f}{\partial t}}_{\Downarrow \; B_t} dt + \frac{\partial f}{\partial B_t} dB_t + \frac{1}{2}\frac{\partial^2 f}{\partial B_t^2} \underbrace{dB_t^2}_{dt}$$

$$= \left(\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial B_t^2} \right) dt + \frac{\partial f}{\partial B_t} \cdot dB_t$$

$df = (B_t \, d$

$+ B_t d$

$$f(t, B_t) = t \cdot B_t \quad (x)$$

$$\boxed{df = t \cdot dB_t + B_t \cdot dt}$$

$$t \cdot B_t = \int_0^T t \cdot dB_t + \int_0^T B_t \cdot dt$$

$$t \cdot B_t - \int_0^T s \cdot dB_s = \boxed{\int_0^T B_s \cdot ds}$$

Focusing

$$\int_0^T B_s \, ds$$

$$\left[\frac{\int_0^T B_s \, dB_s}{\int_0^T t \, dt} \right], \quad \frac{\int_0^T B_t \, dt}{\int_0^T t \, dB_t}$$

$$E\left[\int_0^T B_s\,ds\right]$$

$$= \int_0^T \underbrace{E[B_s]}_{=0}\,ds$$

$$= 0$$

$$V\left[\int_0^T B_s\,ds\right]$$

$$= \boxed{E\left[\left(\int_0^T B_s\,ds\right)^2\right]}$$

$$E\left[\left(\int_0^T B_s \, ds\right)^2\right]$$

$$= E\left[\int_0^T B_s \, ds \cdot \int_0^T B_u \, du\right]$$

$$= E\left[\int_0^T \int_0^T B_u \cdot B_s \cdot du \cdot ds\right]$$

$$= \cancel{\min(u,s)}$$

$$= \int_0^T \int_0^T E[B_u \cdot B_s] \, du \cdot ds$$

$$= \int_0^T \int_s^T s\,du\,ds + \int_0^T \int_0^s u\,du\,ds$$

$u \geq s$ $\qquad$ $u \leq s$

$$= \frac{1}{3} t^3$$

$$\int_0^T B_s\,dt \Rightarrow \boxed{N\left(0, \frac{1}{3}T^3\right)}$$

$$\int_0^T B_t\,dB_t = \frac{1}{2}B_t^2 - \frac{1}{2}T$$

$$\int_0^T t\,dt = \frac{1}{2}T^2$$

$$\int_0^T t\,dB_t \Rightarrow$$