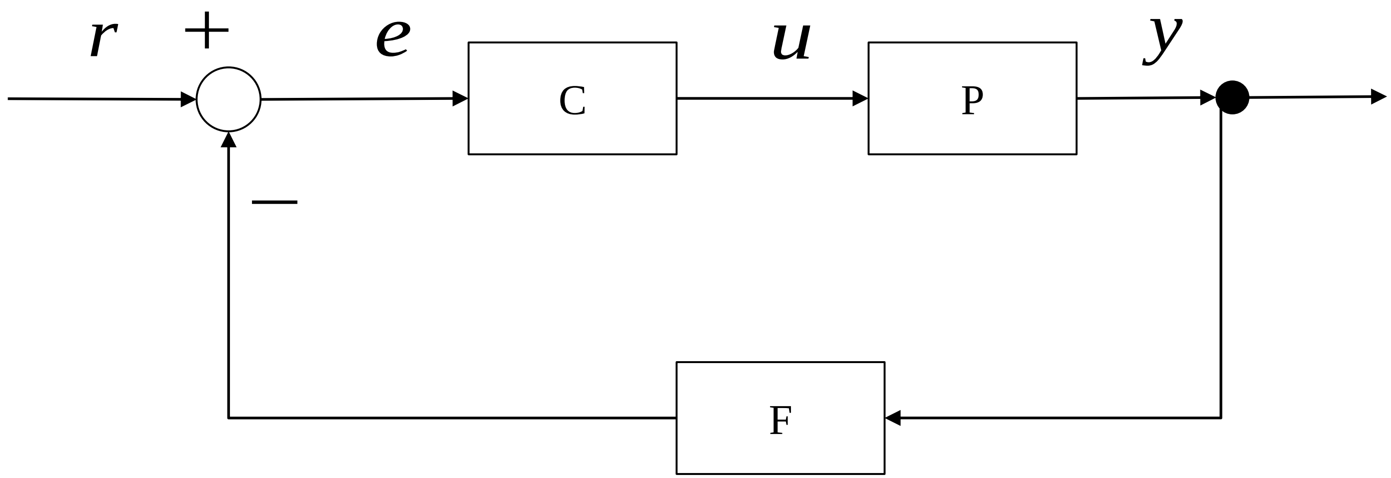
System Optimization, Prediction and Control

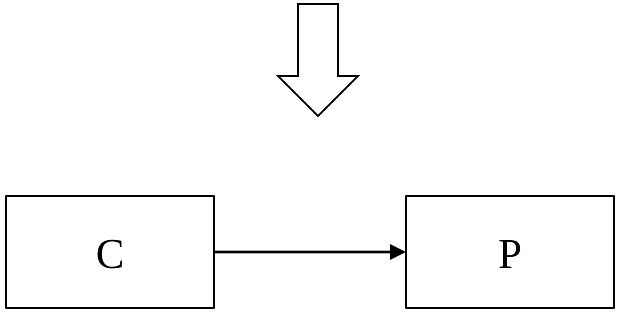


HYUNSOO LEE

Control Theory (1)

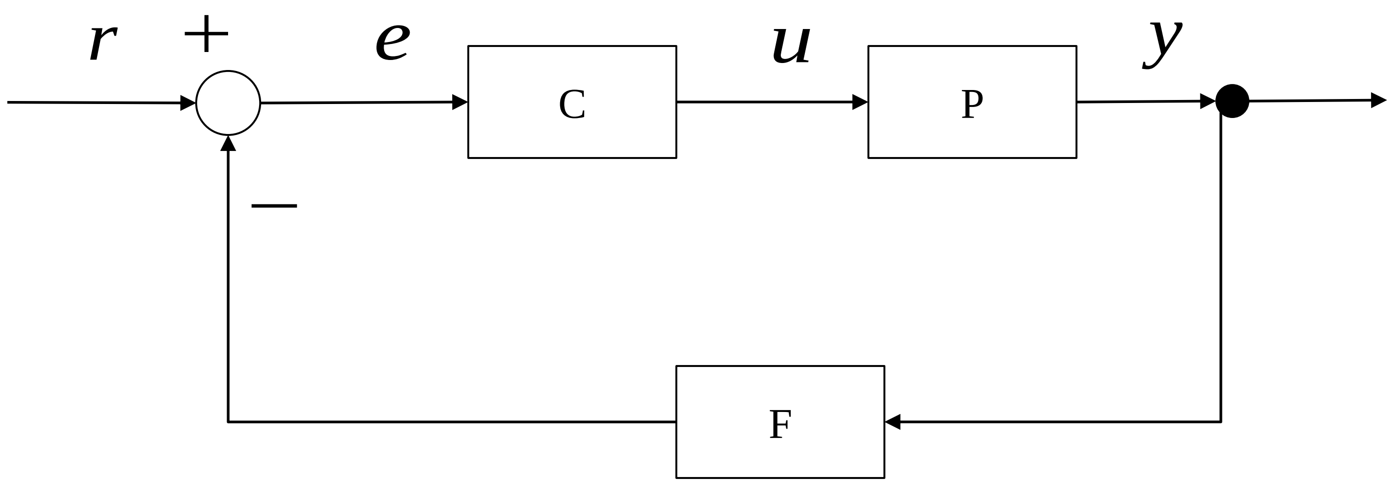
- Control theory

$$y = f(x)$$



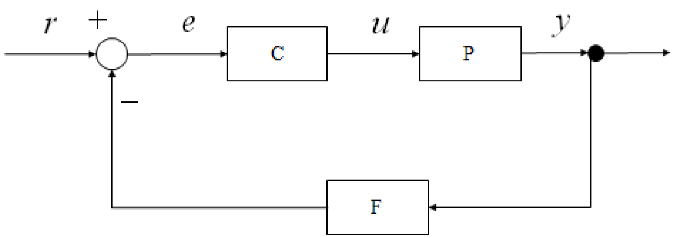
Control Theory (2)

- Closed-loop transfer function



Control Theory (3)

- Interpretation (1)



$$Y(s) = P(s) \cdot U(s)$$

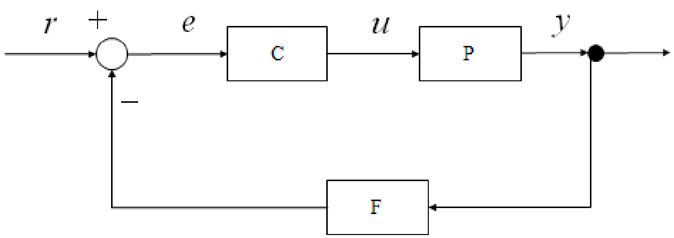
$$U(s) = C(s) \cdot E(s)$$

$$E(s) = R(s) - F(s) \cdot Y(s)$$

$$Y(S) = \left(\frac{P(s) \cdot C(s)}{1 + F(s) \cdot P(s) \cdot C(s)} \right) R(s)$$

Control Theory (4)

- Interpretation (2)
  - Closed-loop transfer function

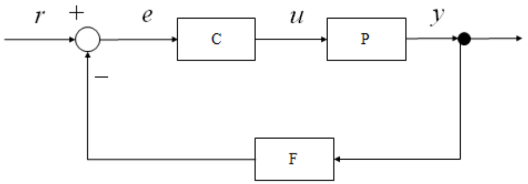


$$Y(S) = \left(\frac{P(s) \cdot C(s)}{1 + F(s) \cdot P(s) \cdot C(s)} \right)$$

$$Y(S) = H(s) R(s)$$

Control Theory (5)

- PID Control



  - Most used control design
  - Proportional-Integral-Differential Control

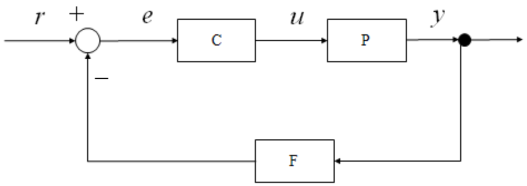
$$u(t) = K_p e(t) + K_I \int e(t)dt + K_I \frac{d}{dt} e(t)$$

  - In case of "MIMO" system

$$u(t) = K_p e(t) + K_I \frac{1}{s} e(t) + K_I t e(t)$$

$$u(t) = \left(K_p + K_I \frac{1}{s} + K_I t \right) e(t)$$

PID Control (1)



- Another control theory →PID Control
  - Most used control design
  - Proportional-Integral-Differential Control

$$U(s) = C(s) \cdot E(s)$$

⇩

$$u(t) = K_p e(t) + K_I \int e(t)dt + K_d \frac{d}{dt} e(t)$$

PID Control (2)

- PID Control

$$u(t) = K_p e(t) + K_I \int e(t)dt + K_d \frac{d}{dt} e(t)$$

⇩

- Role of "P" $K_p e(t)$: Minimize current error
- Role of "I" $K_I \int e(t)dt$: Reflect past error trends
- Role of "D" $K_d \frac{d}{dt} e(t)$: Prevent future error patterns

PID Control (3)

- Rearrangement of "PID" using Laplace transform

$$u(t) = K_p e(t) + K_I \int e(t)dt + K_d \frac{d}{dt} e(t)$$

⇩ Laplace transform

$$u(s) = K_p e(s) + K_I \frac{1}{s} e(s) + K_d s e(s)$$

$$u(s) = \left(K_p + K_I \frac{1}{t} + K_d t \right) e(s)$$

PID Control (4)

- Laplace transform (0)

$$F(s) \triangleright \ell\{f(t)\} = \int_0^\infty |f(t)| e^{-st} dt$$

$$\ell\{1\} = \int_0^\infty |1| e^{-st} dt = \left[-\frac{1}{s} e^{-st} \right]_0^\infty = \frac{1}{s}$$

PID Control (5)

- Laplace transform (1)

$$F(s) \triangleright \ell\{f(t)\} = \int_0^\infty | f(t) | e^{-st} dt$$

$$\ell\{\frac{df}{dt}\} = \int_0^\infty | f'(t) | e^{-st} dt = [e^{-st} f(t)]_0^\infty + s\int_0^\infty | f(t) | e^{-st} dt$$

$$= -f(0) + s\ell\{f(t)\}$$

$$= sf(s)$$

PID Control (6)

- Laplace transform (2-1)

$$\ell\{\int f(t)dt\} =$$

$$F(s)G(s) = \int_0^\infty e^{-st} f(p)dp \int_0^\infty e^{-sk} g(k)dk$$

$$= \int_0^\infty \int_0^\infty e^{-sp} f(p) e^{-sk} g(k) dpdk$$

$$= \int_0^\infty f(p)dp \int_0^\infty e^{-s(p+k)} g(k)dk$$

$$(t = p + k, dt = dk)$$

PID Control (7)

- Laplace transform (2-2)

$$F(s)G(s) = \int_0^\infty f(p)dp \int_0^\infty e^{-s(p+k)} g(k)dk$$

$$= \int_0^\infty e^{-st} dt \int_0^t f(p) g(t-p) dp$$

$$= \int_0^\infty e^{-st} \left\{ \int_0^t f(p) g(t-p) dp \right\} dt = l\{ f * g \}$$

$$l\{ \int_0^t f(t) dt \} = \frac{F(s)}{s}$$

PID Control (8)

- Finally,

$$u(t) = K_p e(t) + K_I \int e(t)dt + K_I \frac{d}{dt} e(t)$$

$$u(s) = K_p e(s) + K_I \frac{1}{t} e(s) + K_I s e(s)$$

$$u(s) = \left(K_p + K_I \frac{1}{s} + K_I s \right) e(s)$$